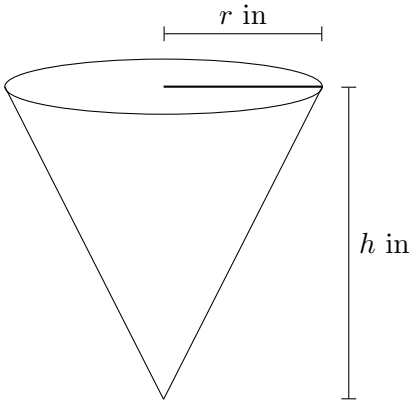
4. [8 points]

Sunny and Tyrell own an ice cream shop together. They want to sell waffle cones in the usual shape of a cone, as shown on the right. The cost, in dollars, of a waffle cone with radius r inches and height h inches is

$$\frac{r}{2}\left(\sqrt{h^2+r^2}\right).$$



Sunny and Tyrell want to spend exactly \$5 on a waffle cone that can fit the most ice cream (i.e has the largest volume).

Note that the volume of a cone of radius r and height h is $\frac{\pi r^2 h}{3}$.

a. [3 points] Write a formula for h in terms of r if the cone costs \$5.

Solution: Because Sunny and Tyrell want to spend exactly \$5 on a waffle cone, we must have $\frac{r}{2}\left(\sqrt{h^2+r^2}\right)=5$. Solving this equation for h, we find

$$\begin{aligned}\sqrt{h^2+r^2}&=\frac{10}{r}\\ h^2+r^2&=\frac{100}{r^2}\\ h^2&=\frac{100}{r^2}-r^2\\ h&=\sqrt{\frac{100}{r^2}-r^2}.\end{aligned}$$

Answer: $h=\sqrt{\frac{100}{r^2}-r^2}$

b. [2 points] Write a formula for the function $V(r)$ which gives the volume, in cubic inches, of an ice cream cone that costs \$5 in terms of r only. *Your formula should not include the letter h.*

Solution: The volume of the ice cream cone is given by $\frac{\pi r^2 h}{3}$. Using our answer from part a., we have

$$V(r)=\frac{\pi r^2\left(\sqrt{\frac{100}{r^2}-r^2}\right)}{3}.$$

Answer: $V(r)=\frac{\pi r^2\left(\sqrt{\frac{100}{r^2}-r^2}\right)}{3}$

c. [3 points] What is the domain of $V(r)$ in the context of this problem?

Solution: Note that r cannot be equal to 0 since then the cost would be 0 rather than \$5, so we know $r>0$.
Also note that $h^2\geq 0$. From part a., we know that $h^2=\frac{100}{r^2}-r^2$, so we have

$$\begin{aligned}\frac{100}{r^2}-r^2&\geq 0\\ 100&\geq r^4\\ 10&\geq |r^2|=r^2\\ \sqrt{10}&\geq |r|=r\ \text{(since } r>0).\end{aligned}$$

Answer: $(0,\sqrt{10})$ **or** $(0,\sqrt{10}]$