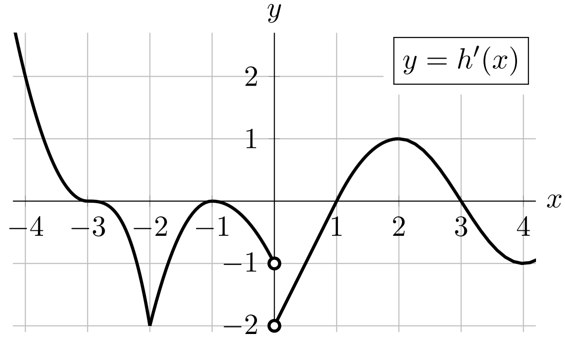
5. [15 points]
Shown on the right is the graph of $h'(x)$, the **derivative** of a function $h(x)$. Assume that h is continuous on its entire domain $(-\infty, \infty)$.

Use this graph to answer the questions below.

You may also use the fact that $\boxed{h(-4) = 5}$.



a. [3 points] Find the linear approximation $L(x)$ of $h(x)$ near $x = -4$, and use your formula to approximate $h(-3.9)$.

Solution:

$$L(x) = h(4) + h'(4)(x - (-4)) = 5 + 2(x + 4)$$

and our linear approximation of $h(-3.9)$ is therefore $h(-3.9) \approx L(-3.9) = 5 + 2(0.1) = 5.2$.

Answer: $L(x) =$ $5 + 2(x + 4)$ and $h(-3.9) \approx$ $5 + 2(0.1) = 5.2$

b. [2 points] Is the estimate of $h(-3.9)$ in part **a.** an overestimate or underestimate of the actual value, or is there not enough information to decide? Briefly explain your reasoning.

Circle one: $\boxed{\text{OVERESTIMATE}}$ UNDERESTIMATE NOT ENOUGH INFORMATION

Brief explanation:

Solution: The second derivative is negative (since $h'(x)$ is decreasing/the slope of $h'(x)$ is negative) on the interval $(-4, -3.9)$ so $h(x)$ is concave down on this interval. Therefore, the tangent line to $y = h(x)$ at $x = -4$ is above the curve $y = h(x)$ at $x = -3.9$ and the resulting linear approximation of $h(-3.9)$ must be an overestimate.

For each question below, circle **all** correct choices. You do not need to justify your answers.

c. [2 points] At which of the following values of x does $h(x)$ have a critical point?

$x = -2$ $\boxed{x = -1}$ $\boxed{x = 0}$ $x = 2$ $\boxed{x = 3}$ NONE OF THESE

d. [2 points] At which of the following values of x does $h(x)$ have a local maximum?

$x = -1$ $x = 0$ $x = 1$ $x = 2$ $\boxed{x = 3}$ NONE OF THESE

e. [2 points] At which of the following values of x does $h(x)$ have an inflection point?

$x = -3$ $\boxed{x = -2}$ $\boxed{x = -1}$ $\boxed{x = 0}$ $\boxed{x = 2}$ NONE OF THESE

f. [2 points] If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the hypotheses of the Mean Value Theorem?

$[-4, -1]$ $[-1, 2]$ $\boxed{[1, 3]}$ $\boxed{[2, 4]}$ NONE OF THESE

g. [2 points]. If $g(x) = h'(x)$, on which of the following interval(s) does $g(x)$ satisfy the conclusion of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $\boxed{[-1, 2]}$ $\boxed{[1, 3]}$ $\boxed{[2, 4]}$ NONE OF THESE