Problem 7. [5 points] A function $g(x)$ is given by the following formula, where $K$ and $M$ are constants:

$$g(x) = \begin{cases} 
Ke^{-x+5} & x \leq 5 \\
M + \sqrt{x+4} & x > 5.
\end{cases}$$

Find all values of $K$ and $M$ so that $g(x)$ is differentiable on $(-\infty, \infty)$. Write NONE if there are no such values. You do not need to simplify your answers, but show your work clearly.

Solution: When $x < 5$, the derivative of $g(x)$ is $g'(x) = -Ke^{-x+5}$. When $x > 5$, the derivative of $g(x)$ is $g'(x) = \frac{1}{2\sqrt{x+4}}$. In order for $g(x)$ to be differentiable on $(-\infty, \infty)$, it must be differentiable at $x = 5$. Therefore, we need the slope to the left and right of $x = 5$ to match. That is, we need $-Ke^{-5+5} = \frac{1}{2\sqrt{5+4}}$.

Solving for $K$, we have

$$K = -\frac{1}{2(3)} = -\frac{1}{6}.$$  

Requiring that $g(x)$ is differentiable at $x = 5$ also requires that $g(x)$ be continuous at $x = 5$. Using this and the value of $K$ we found above to solve for $M$, we have

$$Ke^{-5+5} = M + \sqrt{5+4}$$

$$-\frac{1}{6} = M + 3$$

$$M = -\frac{1}{6} - 3$$

$$M = -\frac{19}{6}.$$

Answer: $K = \frac{-1}{6}$ and $M = \frac{-19}{6}$. 
