7. [5 points] A function $g(x)$ is given by the following formula, where K and M are constants:

$$g(x) = \begin{cases} Ke^{-x+5} & x \leq 5 \\ M + \sqrt{x+4} & x > 5. \end{cases}$$

Find all values of K and M so that $g(x)$ is differentiable on $(-\infty, \infty)$. Write NONE if there are no such values. You do not need to simplify your answers, but show your work clearly.

Solution: When $x < 5$, the derivative of $g(x)$ is $g'(x) = -Ke^{-x+5}$. When $x > 5$, the derivative of $g(x)$ is $g'(x) = \dfrac{1}{2\sqrt{x+4}}$. In order for $g(x)$ to be differentiable on $(-\infty, \infty)$, it must be differentiable at $x = 5$. Therefore, we need the slope to the left and right of $x = 5$ to match. That is, we need

$$-Ke^{-5+5} = \frac{1}{2\sqrt{5+4}}.$$

Solving for K, we have

$$K = -\frac{1}{2(3)} = -\frac{1}{6}.$$

Requiring that $g(x)$ is differentiable at $x = 5$ also requires that $g(x)$ be continuous at $x = 5$. Using this and the value of K we found above to solve for M, we have

$$\begin{aligned} Ke^{-5+5} &= M + \sqrt{5+4} \\ -\frac{1}{6} &= M + 3 \\ M &= -\frac{1}{6} - 3 \\ M &= -\frac{19}{6}. \end{aligned}$$

Answer: $K =$ $-\frac{1}{6}$ and $M =$ $-\frac{19}{6}$