8. [8 points] Prairie dogs named Paws and Dot have been hard at work digging a tunnel. Consider the functions \(L\) and \(C\) defined as follows:
- \(L(w)\) is the length of the tunnel, in feet, when \(w\) pounds of dirt have been removed.
- \(C(w)\) is the total number of Calories the prairie dogs have burned digging their tunnel when they have removed a total of \(w\) pounds of dirt for their tunnel.

The functions \(L(w)\) and \(C(w)\) are both invertible and differentiable.

a. [4 points] Complete the sentence below to give a practical interpretation of the equation 
\[
(L^{-1})'(10) = 24.
\]

*In order to increase the length of the tunnel from 10 feet to 10.25 feet, ...*

\[\text{Solution: the prairie dogs have to remove approximately 6 pounds of dirt.}\]

b. [4 points]

i. Which of the following expressions gives the length, in feet, of the prairie dog tunnel when the prairie dogs have burned a total of \(x\) Calories digging? Circle the one correct expression.

\[
C(L^{-1}(x)) \quad C^{-1}(L(x)) \quad L(C^{-1}(x)) \quad L^{-1}(C(x))
\]

ii. Use the answer you selected in part i to find an expression for the instantaneous rate of change of the length of the prairie dog tunnel, in feet per calorie, when the prairie dogs have burned a total of 2000 calories digging. Simplify as much as possible. Note that your final answer may involve the function names \(L, L^{-1}, L', C, C^{-1},\) and \(C'\) but should not involve the function names \((L^{-1})'\) or \((C^{-1})'\).

\[\text{Solution: We are trying to find a simplified expression for} \ \frac{d}{dx} \left( L \left( C^{-1}(x) \right) \right) \text{ at } x = 2000.\]

*Using the chain rule and the formula for the derivative of the inverse of a function, we have*

\[
\frac{d}{dx} \left( L \left( C^{-1}(x) \right) \right) = L' \left( C^{-1} \left( x \right) \right) \cdot (C^{-1})' \left( x \right)
= L' \left( C^{-1} \left( x \right) \right) \cdot \frac{1}{C' \left( C^{-1} \left( x \right) \right)}.
\]

*So at \(x = 2000\), we have*

\[
L' \left( C^{-1} \left( 2000 \right) \right) \cdot \frac{1}{C' \left( C^{-1} \left( 2000 \right) \right)} = \frac{L' \left( C^{-1} \left( 2000 \right) \right)}{C' \left( C^{-1} \left( 2000 \right) \right)}.
\]

\[\text{Answer:} \ \frac{L' \left( C^{-1} \left( 2000 \right) \right)}{C' \left( C^{-1} \left( 2000 \right) \right)}\]