9. [11 points] A continuous function $w(x)$ and its derivative $w'(x)$ are given by

$$w(x) = \begin{cases} x^2(3x^2+10x-9) & x \le 1 \\ -2\ln(3x-2)+4 & x > 1 \end{cases} \quad \text{and} \quad w'(x) = \begin{cases} 6x(x+3)(2x-1) & x < 1 \\ \dfrac{-6}{3x-2} & x > 1. \end{cases}$$

a. [2 points] Find the x-coordinates of all critical points of $w(x)$. If there are none, write NONE. You do not need to justify your answer.

Solution: The critical points of $w(x)$ occur when $w'(x) = 0$ or $w'(x)$ does not exist. Looking at the formula provided for the derivative, $w'(x) = 0$ when $x = 0, -3$, and $\frac{1}{2}$ and $w'(x)$ does not exist when $x = 1$*. Note that although the denominator of the second piece of w' is 0 when $x = 2/3$, the formula for $w'(x)$ when $x = 2/3$ is given by the first piece. So this is not a critical point.
(*We can also verify that $w'(1)$ is not defined by comparing the values of the first and second pieces of w' at 1.)

Answer: Critical point(s) at $x =$ $-3, 0, \dfrac{1}{2}, 1$

For each part below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

b. [4 points] Find the x-coordinates of all global minima and global maxima of $w(x)$ **on the interval** $(-\infty, 0)$. If there are none of a particular type, write NONE.

Solution: The only critical point of $w(x)$ on $(-\infty, 0)$ is $x = -3$. We have

- $w(-3) = (-3)^2(3(-3)^2 + 10(-3) - 9) = 9(27 - 30 - 9) = 9(-12) = -108$
- $\lim_{x\to-\infty} w(x) = \infty$
- $\lim_{x\to 0^-} w(x) = 0.$

Therefore, there is a global minimum at $x = -3$ and no global maximum.

Answer: Global min(s) at $x =$ -3 and Global max(es) at $x =$ **None**

c. [5 points] Find the x-coordinates of all global minima and global maxima of $w(x)$ **on the interval** $\left[-1, \frac{e+2}{3}\right]$. If there are none of a particular type, write NONE.
In case it is useful, note that $1 < \frac{e+2}{3} < 2$.

Solution: The critical points in $\left[-1, \frac{e+2}{3}\right]$ are $x = 0$, $x = \frac{1}{2}$, $x = 1$.
Note that the Extreme Value Theorem applies here, so we are guaranteed both a global minimum and global maximum. Comparing values of w at critical points and endpoints, we have

- $w(-1) = (-1)^2(3(-1)^2 + 10(-1) - 9) = 3 - 10 - 9 = -16$
- $w(\frac{e+2}{3}) = -2\ln(e + 2 - 2) + 4 = -2\ln(e) + 4 = -2 + 4 = 2$
- $w(0) = 0^2(3(0)^2 + 10(0) - 9) = 0$
- $w(\frac{1}{2}) = (\frac{1}{2})^2(3(\frac{1}{2})^2 + 10(\frac{1}{2}) - 9) = \frac{1}{4}(\frac{3}{4} + 5 - 9) = \frac{1}{4}(\frac{3}{4} - 4) = \frac{3}{16} - 1 = -\frac{13}{16}$
- $w(1) = (1)^2(3(1)^2 + 10(1) - 9) = 3 + 10 - 9 = 4.$

Therefore, there is a global minimum at $x = -1$ and a global maximum at $x = 1$.

Answer: Global min(s) at $x =$ -1 and Global max(es) at $x =$ 1