8. [6 points] The equation \( x^2 + xy + 2y^2 = 28 \) defines \( y \) implicitly as a function of \( x \).
   a. [4 points] Compute \( \frac{dy}{dx} \). Show every step of your work.

   Answer: 

   b. [2 points] Find an equation of the line tangent to the curve defined by \( x^2 + xy + 2y^2 = 28 \) at the point \((2, 3)\).

   Answer: 

9. [6 points] The equation \( x + \frac{1}{3}y^3 - y = 1 \) implicitly defines \( x \) and \( y \) as functions of each other. Implicitly differentiating this equation with respect to \( x \) and solving for \( \frac{dy}{dx} \) gives

   \[
   \frac{dy}{dx} = \frac{-1}{y^2 - 1}.
   \]

   Let \( C \) be the graph of the equation \( x + \frac{1}{3}y^3 - y = 1 \). Note that all points listed as possible answers below do actually lie on the graph \( C \).

   a. [2 points] Circle all points below at which the line tangent to \( C \) is horizontal.

   \(-5, 3\) \hspace{1cm} \left(\frac{1}{3}, -1\right) \hspace{1cm} (1, 0) \hspace{1cm} \left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right) \hspace{1cm} \left(\frac{5}{3}, 1\right) \hspace{1cm} \text{NONE OF THESE}

   b. [2 points] Circle all points below at which the line tangent to \( C \) is vertical.

   \(-5, 3\) \hspace{1cm} \left(\frac{1}{3}, -1\right) \hspace{1cm} (1, 0) \hspace{1cm} \left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right) \hspace{1cm} \left(\frac{5}{3}, 1\right) \hspace{1cm} \text{NONE OF THESE}

   c. [2 points] Circle all points below at which \( \frac{dy}{dx} \) and \( \frac{dx}{dy} \) are equal to each other.

   \(-5, 3\) \hspace{1cm} \left(\frac{1}{3}, -1\right) \hspace{1cm} (1, 0) \hspace{1cm} \left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right) \hspace{1cm} \left(\frac{5}{3}, 1\right) \hspace{1cm} \text{NONE OF THESE}