8. [6 points] The equation $x^2 + xy + 2y^2 = 28$ defines y implicitly as a function of x.

a. [4 points] Compute $\frac{dy}{dx}$. Show every step of your work.

Answer: ______________________________

b. [2 points] Find an equation of the line tangent to the curve defined by $x^2 + xy + 2y^2 = 28$ at the point $(2, 3)$.

Answer: ______________________________

9. [6 points] The equation $x + \frac{1}{3}y^3 - y = 1$ implicitly defines x and y as functions of each other. Implicitly differentiating this equation with respect to x and solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = \frac{-1}{y^2 - 1}.$$

Let $\mathcal{C}$ be the graph of the equation $x + \frac{1}{3}y^3 - y = 1$. *Note that all points listed as possible answers below* <u>*do*</u> *actually lie on the graph* $\mathcal{C}$.

a. [2 points] Circle all points below at which the line tangent to $\mathcal{C}$ is *horizontal*.

$(-5, 3)$ $\left(\frac{1}{3}, -1\right)$ $(1, 0)$ $\left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right)$ $\left(\frac{5}{3}, 1\right)$ NONE OF THESE

b. [2 points] Circle all points below at which the line tangent to $\mathcal{C}$ is *vertical*.

$(-5, 3)$ $\left(\frac{1}{3}, -1\right)$ $(1, 0)$ $\left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right)$ $\left(\frac{5}{3}, 1\right)$ NONE OF THESE

c. [2 points] Circle all points below at which $\frac{dy}{dx}$ and $\frac{dx}{dy}$ are equal to each other.

$(-5, 3)$ $\left(\frac{1}{3}, -1\right)$ $(1, 0)$ $\left(1 + \frac{\sqrt{2}}{3}, \sqrt{2}\right)$ $\left(\frac{5}{3}, 1\right)$ NONE OF THESE