1. [10 points]

A portion of the graph of a function $g(x)$ is shown to the right, along with some values of an invertible, differentiable function $h(x)$ and its derivative $h^{\prime}(x)$ below. Note that:

- $g(x)$ is linear on [3, 5];
- $g(x)$ has a vertical asymptote at $x=-1$.

| $x$ | -2 | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $h(x)$ | -1 | $-e^{-1}$ | 0 | $\sqrt{2}$ | $e$ |
| $h^{\prime}(x)$ | 2 | 1 | $\pi$ | 5 | $\sqrt{3}$ |


a. [2 points] Let $M(x)=x^{2} h(x)$. Find $M^{\prime}(-2)$.

Solution: By the Product Rule, $M^{\prime}(x)=2 x h(x)+x^{2} h^{\prime}(x)$, so

$$
M^{\prime}(-2)=-4 h(-2)+4 h^{\prime}(-2)=(-4)(-1)+4(2)=12
$$

Answer: $\quad M^{\prime}(-2)=$ $\qquad$
b. [2 points] Let $K(x)=\frac{g(x)}{h(x)}$. Find $K^{\prime}(4)$.

Solution: Using the Quotient Rule,

$$
K^{\prime}(4)=\frac{g^{\prime}(4) h(4)-g(4) h^{\prime}(4)}{h(4)^{2}}=\frac{(-1)(\sqrt{2})-(1)(5)}{2}=\frac{-\sqrt{2}-5}{2} .
$$

Answer: $\quad K^{\prime}(4)=\frac{\frac{-\sqrt{2}-5}{2}}{}$
c. [2 points] Find $\left(h^{-1}\right)^{\prime}(0)$.

Solution: Using the rule for the derivative of an inverse function,

$$
\left(h^{-1}\right)^{\prime}(0)=\frac{1}{h^{\prime}\left(h^{-1}(0)\right)}=\frac{1}{h^{\prime}(2)}=\frac{1}{\pi}
$$

Answer: $\left(h^{-1}\right)^{\prime}(0)=\frac{1}{\pi}$
d. [2 points] On which of the following intervals does $g(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct answers.

$$
\left[\begin{array}{lll}
{[-3,-1]} & {[0,2]} & {[1,3]}
\end{array}[2,4] \quad\right. \text { NONE OF THESE }
$$

e. [2 points] On which of the following intervals does $g(x)$ satisfy the conclusion of the Mean Value Theorem? Circle all correct answers.
$[-3,-1]$
$[0,2]$
$[1,3]$
$[2,5]$
NONE OF THESE

