

2. [8 points] Throughout this problem, let $K(x) = e^x - ex$. In case it is helpful, $e \approx 2.7$.
- a. [1 point] Find a formula for $K'(x)$.

Answer: $K'(x) =$ _____ $e^x - e$ _____

- b. [4 points] Find the x -coordinate of all global minimum(s) and global maximum(s) of $K(x)$ **on the interval** $[0, 3]$. If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure that you show enough evidence to justify your conclusions.

Solution: Since $K'(x) = e^x - e$, we have that $K'(x) = 0$ exactly when $x = 1$, which is within our domain. Since $K(x)$ is continuous on $[0, 3]$, it must have a global max and min on $[0, 3]$ by the Extreme Value Theorem, and these must occur at the critical point $x = 1$ or at an endpoint of $[0, 3]$. Since $K(0) = 1$, $K(1) = 0$, and $K(3) = e^3 - 3e = e(e^2 - 3) > 2(2^2 - 3) = 2$, we see that $K(x)$ has a min at $x = 1$ and a max at $x = 3$ on the interval $[0, 3]$.

Answer: Global min(s) at $x =$ _____ **1** _____

Answer: Global max(es) at $x =$ _____ **3** _____

- c. [2 points] Find the linear approximation $L(x)$ of the function $K(x)$ at the point $x = 0$.

Solution: The linear approximation $L(x)$ of the function $K(x)$ at the point $x = 0$ is

$$L(x) = K(0) + K'(0)(x - 0) = 1 + (1 - e)x.$$

Answer: $L(x) =$ _____ $1 + (1 - e)x$ _____

- d. [1 point] If you were to use the linear approximation that you found in part c. to estimate $K(0.1)$, would the approximation give you an *underestimate* or *overestimate* of the true value of $K(0.1)$? Circle the correct answer, or circle NEI if there is not enough information to decide.

UNDERESTIMATE

OVERESTIMATE

NEI

Solution: Since $K''(x) = e^x > 0$ for all x , the function $K(x)$ is concave up everywhere, so any linear approximation to $K(x)$ will always give underestimates.