2. [8 points] Throughout this problem, let \( K(x) = e^x - ex \). In case it is helpful, \( e \approx 2.7 \).

a. [1 point] Find a formula for \( K'(x) \).

Answer: \[ K'(x) = e^x - e \]

b. [4 points] Find the \( x \)-coordinate of all global minimum(s) and global maximum(s) of \( K(x) \) on the interval \([0, 3]\). If there are none of a particular type, write NONE. Use calculus to find your answers, and make sure that you show enough evidence to justify your conclusions.

Solution: Since \( K'(x) = e^x - e \), we have that \( K'(x) = 0 \) exactly when \( x = 1 \), which is within our domain. Since \( K(x) \) is continuous on \([0, 3]\), it must have a global max and min on \([0, 3]\) by the Extreme Value Theorem, and these must occur at the critical point \( x = 1 \) or at an endpoint of \([0, 3]\). Since \( K(0) = 1 \), \( K(1) = 0 \), and \( K(3) = e^3 - 3e = e(e^3 - 3) > 2(2^2 - 3) = 2 \), we see that \( K(x) \) has a min at \( x = 1 \) and a max at \( x = 3 \) on the interval \([0, 3]\).

Answer: Global min(s) at \( x = 1 \)

Answer: Global max(es) at \( x = 3 \)

c. [2 points] Find the linear approximation \( L(x) \) of the function \( K(x) \) at the point \( x = 0 \).

Solution: The linear approximation \( L(x) \) of the function \( K(x) \) at the point \( x = 0 \) is

\[ L(x) = K(0) + K'(0)(x - 0) = 1 + (1 - e)x. \]

Answer: \( L(x) = 1 + (1 - e)x \)

d. [1 point] If you were to use the linear approximation that you found in part c. to estimate \( K(0.1) \), would the approximation give you an underestimate or overestimate of the true value of \( K(0.1) \)? Circle the correct answer, or circle NEI if there is not enough information to decide.

UNDERESTIMATE \hspace{2cm} OVERESTIMATE \hspace{2cm} NEI

Solution: Since \( K''(x) = e^x > 0 \) for all \( x \), the function \( K(x) \) is concave up everywhere, so any linear approximation to \( K(x) \) will always give underestimates.