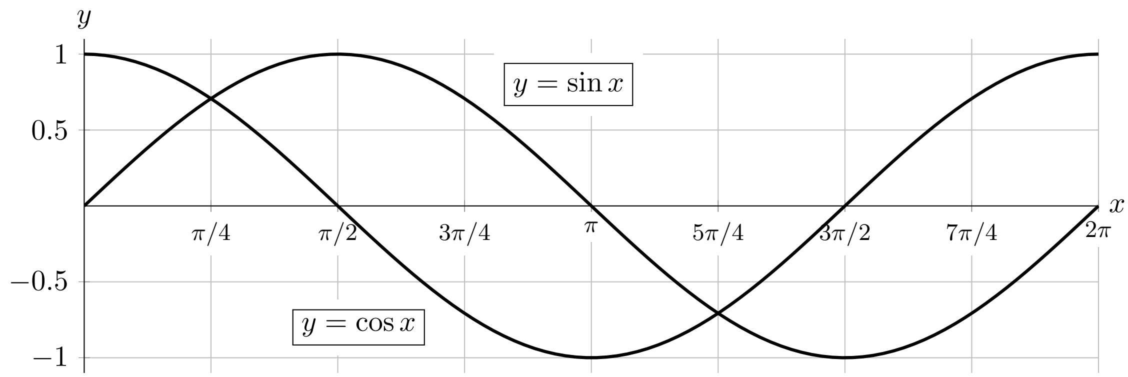
3. [9 points] Throughout this problem, let $f(x) = \sin x + \cos x$. For reference, you may use the graphs of sine and cosine given below, but note that neither of these is a graph of f, since f is their *sum*.



a. [1 point] Give a formula for the derivative of $f(x)$.

Answer: $f'(x) =$ $\cos x - \sin x$

b. [2 points] On which of the following intervals is $f(x)$ increasing? Circle all correct answers.

$\boxed{\left(0, \frac{\pi}{4}\right)}$ $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ $\boxed{\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)}$ $\boxed{\left(\frac{7\pi}{4}, 2\pi\right)}$ NONE OF THESE

c. [2 points] On which of the following intervals is $f(x)$ concave down? Circle all correct answers.

$\boxed{\left(0, \frac{\pi}{4}\right)}$ $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$ $\boxed{\left(\frac{7\pi}{4}, 2\pi\right)}$ NONE OF THESE

d. [4 points] Find and classify all local extrema of $f(x)$ on the interval $(0, 2\pi)$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and show all your work.

Solution: Since $f(x)$ is differentiable and $(0, 2\pi)$ does not contains its endpoints, any local extremum of $f(x)$ on $(0, 2\pi)$ must occur at a point where $f'(x) = 0$, that is, where $\cos(x) = \sin(x)$. From the graph we see that this happens only at $x = \pi/4$ and $x = 5\pi/4$. Since $f''(x) = -\sin x - \cos x = -f(x)$, from the graph we see that $f''\left(\frac{\pi}{4}\right) < 0$ and $f''\left(\frac{5\pi}{4}\right) > 0$, so $\frac{\pi}{4}$ is a local max and $\frac{5\pi}{4}$ is a local min by the Second Derivative Test.

Answer: Local min(s) at $x =$ $\frac{5\pi}{4}$ and Local max(es) at $x =$ $\frac{\pi}{4}$