4. [10 points] Octavia is now inflating her rear bicycle tire with her new air compressor. Let $V(t)$ be the total volume of air, in cubic inches, that the compressor has pumped into the tire t seconds after the compressor has been switched on, and let $P(s)$ be the air pressure, in pounds per square inch (psi), inside the tire when it has been filled with s cubic inches of air.

Octavia has learned that increasing the pressure in her tire will reduce the *rolling resistance* of the tire, which is the energy that is lost, per unit of time, while the tire is rolling. Let $R(p)$ be the rolling resistance, measured in watts, of the tire when the air pressure inside the tire is p psi. Assume the functions V, P, and R are invertible and differentiable.

a. [2 points] Write a number in the blank below to give a practical interpretation of the equation

$$(R^{-1})'(43) = -19.$$

If Octavia wants to reduce the rolling resistance of her tire from 43 to 41 watts, she should increase her tire pressure by about 38 *psi.*

b. [2 points] Circle the **one** equation below that best represents the statement: "If Octavia wants the rolling resistance of her rear wheel to be 32 watts, she needs to run the air compressor for 30 seconds."

(i) $R^{-1}\big(P^{-1}(V^{-1}(32))\big) > 30$ (ii) $P\big(V(30)\big) = R^{-1}(32)$

(iii) $R'\big(P'(V'(30))\big) = 32$ (iv) $R(32) < P\big(V(30)\big)$

c. [2 points] Write a mathematical equation involving a derivative that has the following practical interpretation: "If Octavia increases her tire pressure from 50 psi to 60 psi, she will reduce the tire's rolling resistance by about a half a watt.

Answer: $R'(50) = -\frac{1}{20}$

d. [2 points] Octavia knows that as she increases her tire pressure, the corresponding reduction in rolling resistance decreases as the tire inflates. Therefore the graph of $R(p)$ is:

(i) increasing and concave up (ii) increasing and concave down

(iii) decreasing and concave up (iv) decreasing and concave down

e. [2 points] After experimenting and doing some research, Octavia concludes that V and P are closely modeled by the equations

$$V(t) = \frac{20}{3}t \qquad \text{and} \qquad P(s) = \frac{3}{10}s + 30,$$

and for values of p between 40 and 100 psi, $R'(p)$ is about -0.05. Given this, estimate the rate at which Octavia is reducing her rolling resistance by inflating her tire, in watts per second, when the compressor has been running for 30 seconds. *Show your work.*

Solution: The rate at which Octavia is reducing her rolling resistance is $\frac{dR}{dt}$. Writing $s = V(t)$ and $p = P(s)$ and applying the Chain Rule gives us

$$\frac{dR}{dt} = \frac{dR}{dp} \cdot \frac{dp}{ds} \cdot \frac{ds}{dt} = R'(p) \cdot \frac{3}{10} \cdot \frac{20}{3} = 2R'(p).$$

Therefore, from the estimate $R'(p) \approx -0.05$, we conclude $\frac{dR}{dt}\big|_{t=30} = -0.1$ watts per second.

Answer: 0.1 **watts per second**