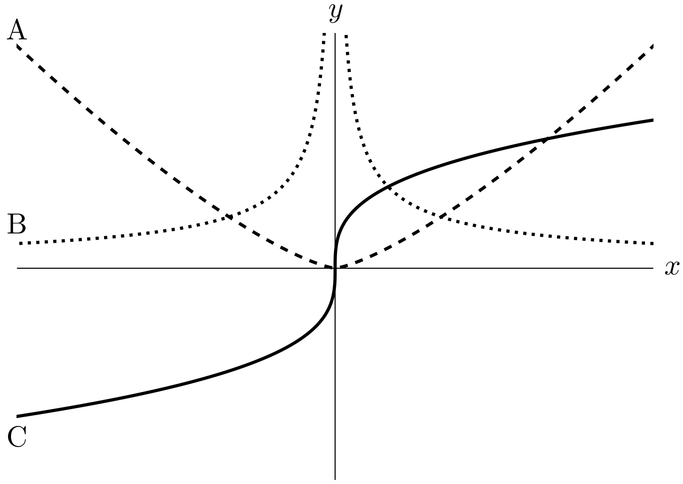
5. [4 points] Shown below are portions of the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Note that the dotted graph has a vertical asymptote at $x = 0$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x)$: A

$f'(x)$: C

$f''(x)$: B

6. [7 points] The function $q(x)$ is given by the following formula, where c and m are constants:

$$q(x) = \begin{cases} c - 4x - x^2 & -3 \le x \le 0 \\ mx & 0 < x \le 2. \end{cases}$$

a. [4 points] Assuming $c = -3$ and $m = 2$, find the x-values of all global minima and global maxima of $q(x)$ on the interval $[-3, 2]$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and show your work.

Solution: On $(-3, 0)$ we have $q'(x) = -4 - 2x$, so the only critical point of $q(x)$ on $(-3, 0)$ is $x = -2$. Since q is continuous on $[-3, 0]$ and we have $q(-3) = 0$, $q(-2) = 1$, and $q(0) = -3$, we see that, on the interval $[-3, 0]$, $q(x)$ has a max of 1 at -2 and a min of -3 at 0.

Now we check $(0, 2]$. On $(0, 2]$, $q(x)$ is linear with slope 2, so the max value of $q(x)$ on $(0, 2]$ is $q(2) = 4$, and we have $q(x) > \lim_{x \to 0^+} q(x) = 0$ for all $0 < x \le 2$.

Putting this all together, we find that on the interval $[-3, 2]$, the function $q(x)$ has a global maximum of 4 at $x = 2$, and a global minimum of -3 at $x = 0$.

Answer: Global min(s) at $x =$ 0 and Global max(es) at $x =$ 2

b. [3 points] Find one pair of values for c and m such that $q(x)$ is differentiable at $x = 0$. Show your work.

Solution: To be differentiable at $x = 0$, the function $q(x)$ must be continuous at 0, which means

$$c - 4 \cdot 0 - 0^2 = m \cdot 0,$$

that is, $c = 0$. But we will also need the derivative of $c - 4x - x^2$ at zero to equal the derivative of mx at zero. Thus $-4 - 2 \cdot 0 = m$, so $m = -4$.

Answer: $c =$ 0 and $m =$ -4