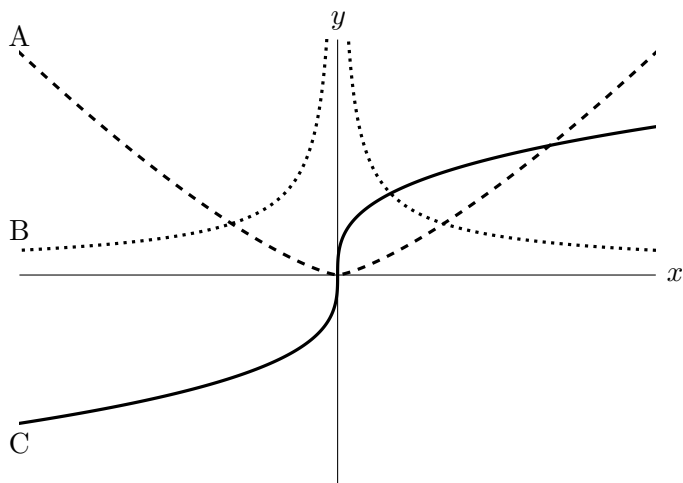


5. [4 points] Shown below are portions of the graphs of  $y = f(x)$ ,  $y = f'(x)$ , and  $y = f''(x)$ . Note that the dotted graph has a vertical asymptote at  $x = 0$ . Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



**Answer:**  $f(x)$  :     A      
 $f'(x)$  :     C      
 $f''(x)$  :     B    

6. [7 points] The function  $q(x)$  is given by the following formula, where  $c$  and  $m$  are constants:

$$q(x) = \begin{cases} c - 4x - x^2 & -3 \leq x \leq 0 \\ mx & 0 < x \leq 2. \end{cases}$$

- a. [4 points] Assuming  $c = -3$  and  $m = 2$ , find the  $x$ -values of all global minima and global maxima of  $q(x)$  on the interval  $[-3, 2]$ . If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and show your work.

*Solution:* On  $(-3, 0)$  we have  $q'(x) = -4 - 2x$ , so the only critical point of  $q(x)$  on  $(-3, 0)$  is  $x = -2$ . Since  $q$  is continuous on  $[-3, 0]$  and we have  $q(-3) = 0$ ,  $q(-2) = 1$ , and  $q(0) = -3$ , we see that, on the interval  $[-3, 0]$ ,  $q(x)$  has a max of 1 at  $-2$  and a min of  $-3$  at 0.

Now we check  $(0, 2]$ . On  $(0, 2]$ ,  $q(x)$  is linear with slope 2, so the max value of  $q(x)$  on  $(0, 2]$  is  $q(2) = 4$ , and we have  $q(x) > \lim_{x \rightarrow 0^+} q(x) = 0$  for all  $0 < x \leq 2$ .

Putting this all together, we find that on the interval  $[-3, 2]$ , the function  $q(x)$  has a global maximum of 4 at  $x = 2$ , and a global minimum of  $-3$  at  $x = 0$ .

**Answer:** Global min(s) at  $x =$      0     and Global max(es) at  $x =$      2    

- b. [3 points] Find one pair of values for  $c$  and  $m$  such that  $q(x)$  is differentiable at  $x = 0$ . Show your work.

*Solution:* To be differentiable at  $x = 0$ , the function  $q(x)$  must be continuous at 0, which means

$$c - 4 \cdot 0 - 0^2 = m \cdot 0,$$

that is,  $c = 0$ . But we will also need the derivative of  $c - 4x - x^2$  at zero to equal the derivative of  $mx$  at zero. Thus  $-4 - 2 \cdot 0 = m$ , so  $m = -4$ .

**Answer:**  $c =$      0     and  $m =$     -4