

3. [9 points] Suppose

$$g(x) = \sqrt{x^2 + 1} \quad \text{and} \quad h(x) = ke^{2x} \ln x,$$

where k is a real number constant. Note that

$$g'(x) = \frac{x}{\sqrt{x^2 + 1}} \quad \text{and} \quad g''(x) = (x^2 + 1)^{-3/2}.$$

- a. [2 points] Find a formula for the linear approximation $L(x)$ of the function $g(x)$ at the point $x = -1$. Your answer should not include the letter g , but you do not need to simplify.

Answer: $L(x) =$ _____

- b. [1 point] Does $L(x)$ give an overestimate or underestimate for $g(x)$ near $x = -1$? Circle your answer below. *No justification needed.*

UNDERESTIMATE

OVERESTIMATE

- c. [3 points] Find a formula for $h'(x)$. Your answer may include the constant k .

Answer: $h'(x) =$ _____

- d. [3 points] Find a value of k for which the function

$$f(x) = \begin{cases} g(x) & x \leq 1 \\ h(x) & x > 1 \end{cases}$$

is differentiable, if this is possible. If no such value of k exists, write DNE on the answer line and briefly justify your answer. *Show your work.* [Note: Recall that $\ln(1) = 0$.]

Answer: $k =$ _____