7. [12 points] Let $f(x) = x^4e^x + 4e$ and $g(x) = -x^2 + (2 + 5e)x - 1$, and let $h(x)$ be the piecewise function

$$h(x) = \begin{cases} f(x) & x \leq 1 \\ g(x) & x > 1. \end{cases}$$

Note that $f(1) = 5e = g(1)$ and $f'(1) = 5e = g'(1)$, so $h(x)$ is continuous and differentiable at $x = 1$. To answer the questions below, you may use the following:

$$f'(x) = x^3e^x(x + 4) \qquad \text{and} \qquad f''(x) = x^2e^x(x + 2)(x + 6).$$

a. [3 points] Find all critical points of $h(x)$. *No justification necessary.*

Answer: $h(x)$ has critical points at $x =$ _____

b. [3 points] Find all critical points of $h'(x)$. *No justification necessary.*

Answer: $h'(x)$ has critical points at $x =$ _____

c. [6 points] Find all inflection points of $h(x)$. *Show all your work. Be sure you show enough evidence to justify your conclusions.*

Answer: $h(x)$ has inflection points at $x =$ _____