

9. [7 points] Alana heats water using her “Simone Steamer,” a peculiar kettle with the face of comic book writer Gail Simone on it, and tracks the depth of water as she fills it up.

a. [2 points] The depth of the water  $H$ , in cm,  $t$  seconds after Alana starts filling the kettle is given by  $H = f(t)$ . Circle the **one** statement below that is best supported by the equation

$$(f^{-1})'(5) = 2.$$

- i. After Alana has been filling the kettle for 5 seconds, the depth of water will increase by about 1 cm in the next half-second.
- ii. It takes approximately one second for the depth of water to increase from 4.5 to 5cm.
- iii. Every two seconds, the depth of water increases by about 5cm.
- iv. After Alana has been filling the kettle for 2 seconds, the depth of water is roughly 5cm.

b. [2 points] The volume  $V$ , in  $\text{cm}^3$ , of water in the kettle is related to the depth  $H$  by the formula

$$V = \frac{1}{5} (H^3 + 4H^2 + 10H).$$

Find an expression for  $\frac{dV}{dt}$  in terms of  $H$  and  $\frac{dH}{dt}$ .

**Answer:**  $\frac{dV}{dt} =$  \_\_\_\_\_

c. [3 points] When Alana has been filling the kettle for 3 seconds, the depth of the water is 5cm. Use this and the fact (from part a.) that  $(f^{-1})'(5) = 2$  to determine the rate at which the **volume** of water in the kettle is increasing at  $t = 3$ . *Show all your work.*

**Answer:** The rate is: \_\_\_\_\_  $\text{cm}^3$  per second.