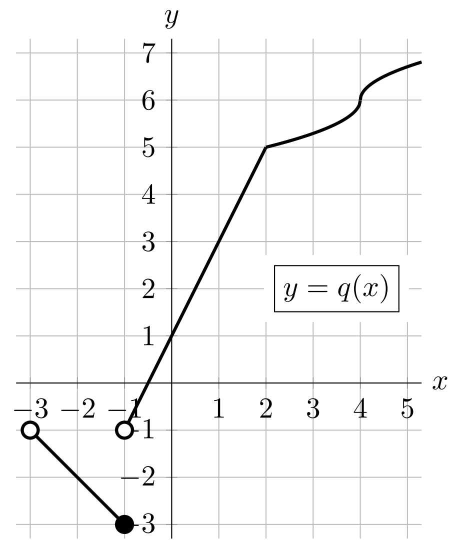
1. [9 points]

A portion of the graph of the invertible function $q(x)$ is shown to the right. Note that:

- $q(x)$ is linear on $(-3,-1]$ and on $(-1,2]$.
- There is a corner at $x=2$.
- The tangent line to $q(x)$ at $x=4$ is vertical.

For parts **a.–c.**, find the **exact** values, or write NEI if there is not enough information to do so, or write DNE if the value does not exist. Your answers should not include the letter q, but you do not need to simplify. *Show work.*



a. [2 points] Let $A(x)=q^{-1}(x)$. Find $A'(4)$.

Solution: $A'(4) = \left(q^{-1}\right)'(4) = \dfrac{1}{q'\left(q^{-1}(4)\right)} = \dfrac{1}{q'(1.5)} = \dfrac{1}{2}.$

Answer: $A'(4) =$ 1/2

b. [2 points] Let $B(x)=\dfrac{x}{q(x)}$. Find $B'(-2)$.

Solution: By the Quotient Rule, $B'(x) = \dfrac{q(x)-xq'(x)}{q(x)^2}$, so

$$B'(-2) = \frac{q(-2)+2q'(-2)}{q(-2)^2} = \frac{-2+2(-1)}{(-2)^2} = \frac{-4}{4} = -1.$$

Answer: $B'(-2) =$ -1

c. [3 points] Let $C(x)=\cos\left(\frac{\pi}{2}xq(x)\right)$. Find $C'(1)$.

Solution: Using the Chain Rule and Product Rule, we have

$$C'(x) = -\sin\left(\frac{\pi}{2}xq(x)\right)\cdot\left(\frac{\pi}{2}q(x)+\frac{\pi}{2}xq'(x)\right).$$

Since $q(1)=3$ and $q'(1)=2$, this means

$$C'(1) = -\sin\left(\frac{3\pi}{2}\right)\cdot\left(\frac{3\pi}{2}+\pi\right) = -\frac{5\pi}{2}\sin\left(\frac{3\pi}{2}\right) = \frac{5\pi}{2}.$$

Answer: $C'(1) =$ $\dfrac{5\pi}{2}$

d. [2 points] On which of the following intervals does $q(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all correct answers. You do not need to show work for this part.

$[-1,1]$ $\quad$ $\boxed{[2,3.5]}$ $\quad$ $[3,5]$ $\quad$ NONE OF THESE

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