10. [9 points] A table of some values of the function $f(x)$ and its first and second derivatives is given below. The functions $f(x)$, $f'(x)$, and $f''(x)$ are continuous everywhere.

x	-4	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-0.6	0	-0.3	-2	-3	-1	0	3	88	204
$f'(x)$	3	0	-1	-2	0	2	0	9	80	0
$f''(x)$	-8	0	-2	0	4	0	0	22	0	-74

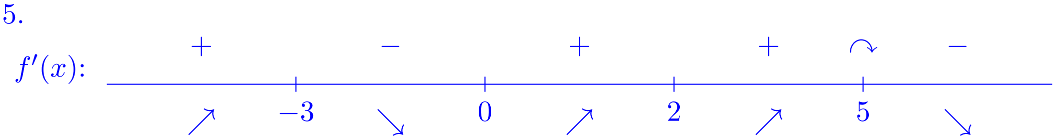
Assume that the critical points of $f(x)$ and $f'(x)$ are as follows, with no additional critical points besides those listed:

$$\text{critical points of } f(x): \quad -3,\ 0,\ 2,\ 5$$

$$\text{critical points of } f'(x): \quad -3,\ -1,\ 1,\ 2,\ 4$$

a. [4 points] Find all local extrema of $f(x)$, and classify each as a max or a min. If there are none of a particular type, write NONE. *No justification is necessary, although limited partial credit may be awarded for work shown.*

Solution: Using the table, we can produce a sign chart for $f'(x)$ as follows. Note that in order to determine the sign of $f'(x)$ on $(5, \infty)$, we can apply the Second Derivative Test at $x = 5$ to conclude that the graph of $f(x)$ is concave down near $x = 5$, so $f(x)$ has a local max at $x = 5$, so $f'(x) < 0$ for $x > 5$.



From this sign chart for $f'(x)$, the First Derivative Test tells us that $f(x)$ has a local min at $x = 0$, and local maxes as $x = -3$ and $x = 5$.

Answer: Local min(s) at $x =$ 0

Answer: Local max(es) at $x =$ -3, 5

b. [3 points] Find all global extrema of $f(x)$ on the interval $[-4, 3]$, and classify each as a max or a min. If there are none of a particular type, write NONE. *No justification is necessary, although limited partial credit may be awarded for work shown.*

Solution: We just need to check the values of $f(x)$ at the two endpoints and at all the critical points of f in the interval $(-4, 3)$. The largest of these values will be the global max of $f(x)$ on $[-4, 3]$, and the least will be the global min. We have:

$$f(-4) = -0.6, \quad f(-3) = 0, \quad f(0) = -3, \quad f(2) = 0, \quad f(3) = 3.$$

So $f(x)$ has a global min at $x = 0$ and a global max at $x = 3$ on the interval $[-4, 3]$.

Answer: Global min(s) at $x =$ 0

Answer: Global max(es) at $x =$ 3

c. [2 points] Circle all intervals below on which $f(x)$ must be <u>concave down</u> on the entire interval.

[$(-4, -2)$] **[$(-3, -1)$]** $(-1, 0)$ **[$(4, 5)$]** **[$(5, \infty)$]** NONE OF THESE