3. [9 points] Suppose

$$g(x) = \sqrt{x^2+1} \qquad \text{and} \qquad h(x) = ke^{2x}\ln x,$$

where k is a real number constant. Note that

$$g'(x) = \frac{x}{\sqrt{x^2+1}} \qquad \text{and} \qquad g''(x) = (x^2+1)^{-3/2}.$$

a. [2 points] Find a formula for the linear approximation $L(x)$ of the function $g(x)$ at the point $x = -1$. Your answer should not include the letter g, but you do not need to simplify.

Solution:

$$L(x) \;=\; g(-1) + g'(-1)(x+1) \;=\; \sqrt{2} - \frac{1}{\sqrt{2}}(x+1).$$

Answer: $L(x) =$ $\sqrt{2} - \frac{1}{\sqrt{2}}(x+1)$

b. [1 point] Does $L(x)$ give an overestimate or underestimate for $g(x)$ near $x = -1$? Circle your answer below. *No justification needed.*

[UNDERESTIMATE] OVERESTIMATE

c. [3 points] Find a formula for $h'(x)$. Your answer may include the constant k.

Solution: Using the Product Rule and the Chain Rule, we get:

Answer: $h'(x) =$ $2ke^{2x}\ln x + \frac{ke^{2x}}{x}$

d. [3 points] Find a value of k for which the function

$$f(x) = \begin{cases} g(x) & x \le 1 \\ h(x) & x > 1 \end{cases}$$

is differentiable, if this is possible. If no such value of k exists, write DNE on the answer line and briefly justify your answer. *Show your work.* [Note: Recall that $\ln(1) = 0$.]

Solution: Since $g(1) = \sqrt{2}$ and $h(1) = ke^2 \ln 1 = 0$, we have $g(1) \neq h(1)$ no matter what k is, so there is no value of k that makes $f(x)$ continuous at $x = 1$, and therefore no value of k that makes $f(x)$ differentiable at $x = 1$.

Answer: $k =$ **DNE**