**5.** [4 points] One of Alana's interns suggests that instead of trying to fix the cost of their Stan Lee cups and maximize the volume, they should instead fix the volume and try to minimize the cost. Assuming they try to match their competitor's standard cup size of 500ml, their cost of producing each cup is

$$C(r) \;=\; \frac{2000}{r} + 5\pi r^2 \quad \text{dollars},$$

where $r$ is the radius of the cup. Assuming the only constraint on $r$ is that $r > 0$, what is the cup radius that minimizes their cost? *Show all your work.*

*Solution:* We need to minimize $C(r)$ over the domain $(0, \infty)$. Taking a derivative, we find

$$C'(r) \;=\; -2000r^{-2} + 10\pi r,$$

so $C(r)$ is differentiable everywhere on $(0, \infty)$. Setting $C'(r) = 0$ and solving we get:

$$\begin{aligned} \frac{2000}{r^2} &= 10\pi r, \\ 2000 &= 10\pi r^3, \\ \frac{200}{\pi} &= r^3, \end{aligned}$$

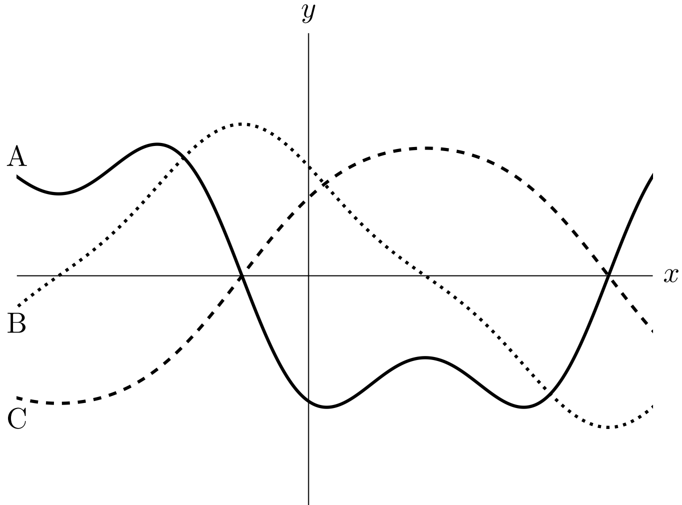
so the only critical point of $C(r)$ on $(0, \infty)$ is $r = \sqrt[3]{\frac{200}{\pi}} = \left(\frac{200}{\pi}\right)^{1/3}$. Since

$$\lim_{x \to 0^+} C(r) \;=\; \infty \;=\; \lim_{x \to \infty} C(r),$$

this critical point must indeed be the global minimum of $C(r)$ on $(0, \infty)$.

**Answer:** $r =$ $\left(\frac{200}{\pi}\right)^{1/3}$ centimeters

**6.** [4 points] Shown below are portions of the graphs of the functions $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.




**Answer:** $f(x)$ : C

$f'(x)$ : B

$f''(x)$ : A