9. [7 points] Alana heats water using her "Simone Steamer," a peculiar kettle with the face of comic book writer Gail Simone on it, and tracks the depth of water as she fills it up.

a. [2 points] The depth of the water H, in cm, t seconds after Alana starts filling the kettle is given by $H = f(t)$. Circle the **one** statement below that is best supported by the equation

$$(f^{-1})'(5) = 2.$$

i. After Alana has been filling the kettle for 5 seconds, the depth of water will increase by about 1 cm in the next half-second.

ii. It takes approximately one second for the depth of water to increase from 4.5 to 5cm.

iii. Every two seconds, the depth of water increases by about 5cm.

iv. After Alana has been filling the kettle for 2 seconds, the depth of water is roughly 5cm.

b. [2 points] The volume V, in cm^3, of water in the kettle is related to the depth H by the formula

$$V = \frac{1}{5}\left(H^3 + 4H^2 + 10H\right).$$

Find an expression for $\frac{dV}{dt}$ in terms of H and $\frac{dH}{dt}$.

Solution: Implicitly differentiating each side with respect to t, we get

$$\frac{dV}{dt} = \frac{1}{5}\left(3H^2\frac{dH}{dt} + 8H\frac{dH}{dt} + 10\frac{dH}{dt}\right) = \left(\frac{3}{5}H^2 + \frac{8}{5}H + 2\right)\frac{dH}{dt}.$$

Answer: $\frac{dV}{dt} =$ $\left(\frac{3}{5}H^2 + \frac{8}{5}H + 2\right)\frac{dH}{dt}$

c. [3 points] When Alana has been filling the kettle for 3 seconds, the depth of the water is 5cm. Use this and the fact (from part **a.**) that $(f^{-1})'(5) = 2$ to determine the rate at which the **volume** of water in the kettle is increasing at $t = 3$. *Show all your work.*

Solution: We must find $\frac{dV}{dt}$ when $t = 3$. We can find this using our answer to part **b.**, provided we can find H and $\frac{dH}{dt}$ when $t = 3$. We are given that the depth of the water when $t = 3$ is $H = f(3) = 5$ cm. And when $t = 3$, the rate $\frac{dH}{dt}$ at which the depth of the water is changing is

$$f'(3) = \frac{1}{(f^{-1})'(f(3))} = \frac{1}{(f^{-1})'(5)} = \frac{1}{2}.$$

So, when $t = 3$, we have

$$\frac{dV}{dt} = \left(\frac{3}{5}(5)^2 + \frac{8}{5}(5) + 2\right)\cdot\frac{1}{2} = \frac{25}{2}.$$

Answer: The rate is: 25/2 cm^3 per second.