- **9.** [11 points] A spherical balloon begins to inflate with air at time t = 0, after which time its radius r, volume V, and surface area A increase. Recall that the volume V and surface area A of a sphere of radius r are given by $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.
 - **a**. [5 points] At what rate is air being blown into the balloon when the balloon's radius is 10 cm and its radius is growing at a rate of 2 cm per second? *Include units*.

Answer:

Suppose the volume V and surface area A of the balloon t seconds after it begins to inflate are given by V = f(t) and A = g(t). These functions are invertible, and the function h(V) defined by $h(V) = g(f^{-1}(V))$ gives the balloon's surface area as a function of its volume.

b. [3 points] Using the given table of values, find h'(8). Your answer must be a *number*, but need not be simplified.

| t | 1 | 8 |
|-------|-------|-------|
| r | 1.24 | 2.48 |
| f(t) | 8 | 64 |
| g(t) | 19.34 | 77.38 |
| dr/dt | 0.41 | 0.10 |
| f'(t) | 8 | 8 |
| g'(t) | 12.90 | 6.45 |

Answer: $h'(8) = _$

c. [3 points] Circle the <u>one</u> statement below that is best supported by the equation

$$(h^{-1})'(50) = \frac{1}{4}.$$

- i. When the balloon's surface area is 50 cm^2 , its volume is 0.25 cm^3 .
- ii. When the balloon's surface area is 50 cm², its surface area is increasing at about $\frac{1}{4}$ the rate at which its volume is increasing.
- iii. When the balloon's surface area is 52 cm^2 , the balloon is about one cubic centimeter larger in volume than it was when its surface area was 48 cm^2 .
- iv. The balloon's surface area increases by about 0.25 $\rm cm^2$ during the time when its volume increases from 50 to 51 $\rm cm^3.$
- v. The balloon's volume increases by about 4 $\rm cm^3$ during the time when its surface area increases from 50 to 51 $\rm cm^2.$