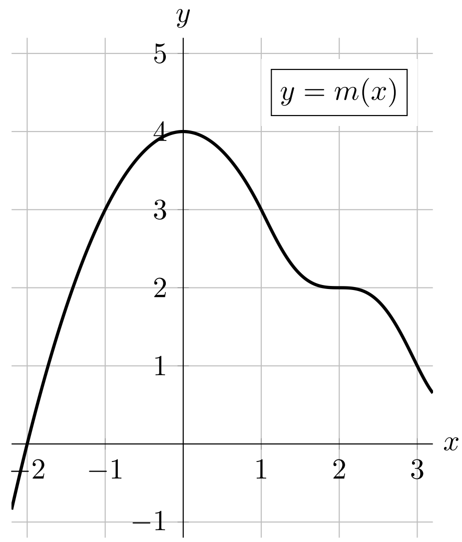
1. [9 points]

A portion of the graph of the function $m(x)$, which is defined for all real numbers, is shown to the right. You are also given the following about $m(x)$:

- $m(x)$ is differentiable everywhere, and has a horizontal tangent line at $x = 2$.
- $m(x) = -x^2 + 4$ for all $x \le 0$.
- The line $y = 5 - 2x$ is tangent to $m(x)$ at $x = 1$.



For parts **a.–c.**, find the **exact** values, or write NEI if there is not enough information to do so, or write DNE if the value does not exist. Your answers should not include the letter m, but you do not need to simplify. *Show work.*

a. [3 points] Let $A(x) = \ln\big(m(x) + x\big)$. Find $A'(-1)$.

Solution: From the graph we see $m(-1) = 3$, and since $m(x) = -x^2 + 4$ for $x \le 0$, we have $m'(-1) = -2(-1) = 2$. Therefore, by the Chain Rule and the rule for differentiating the natural log function, we have

$$A'(x) = \frac{m'(x) + 1}{m(x) + x}, \quad \text{so} \quad A'(-1) = \frac{m'(-1) + 1}{m(-1) - 1} = \frac{2 + 1}{3 - 1} = \frac{3}{2}.$$

Answer: $A'(-1) =$ 3/2

b. [3 points] Let $B(x) = x^3 m(x)$. Find $B'(1)$.

Solution: From the graph we see $m(1) = 3$, and since the line $y = 5 - 2x$ is tangent to $m(x)$ at $x = 1$, we have $m'(1) = -2$. Therefore, by the Product Rule we have

$$B'(x) = 3x^2 m(x) + x^3 m'(x), \quad \text{so} \quad B'(1) = 3m(1) + m'(1) = 9 + (-2) = 7.$$

Answer: $B'(1) =$ 7

c. [3 points] Let $C(x) = \dfrac{m(x)}{x^2}$. Find $C'(2)$.

Solution: From the graph we see $m(2) = 2$ and $m'(2) = 0$, so by the Quotient Rule we have

$$C'(x) = \frac{m'(x)x^2 - 2xm(x)}{x^4}, \quad \text{so} \quad C'(2) = \frac{4m'(2) - 4m(2)}{2^4} = \frac{0 - 4(2)}{16} = -\frac{1}{2}.$$

Answer: $C'(2) =$ $-1/2$