Problem 5 continues from the previous page. Recall that

$$f(x) = \begin{cases} g(x) & x \le 1 \\ h(x) & x > 1 \end{cases}$$

and $L(x)$ is the linear approximation of $g(x)$ at $x = 1$. For part **d.** below, let C and k be the constants that you found in part **c.**, so $f(x)$ is continuous and differentiable.

d. [4 points] You are given that $g''(x) > 0$ on the domain of $g(x)$, while $h''(x) < 0$ on the domain of $h(x)$. Using this, answer the questions below, and *justify each answer with a brief explanation.*

i. Does the function $L(x)$ from part **b.** give an overestimate or underestimate for $g(x)$ near $x = 1$? Circle your answer, and briefly justify it.

$\boxed{\text{UNDERESTIMATE}}$ OVERESTIMATE

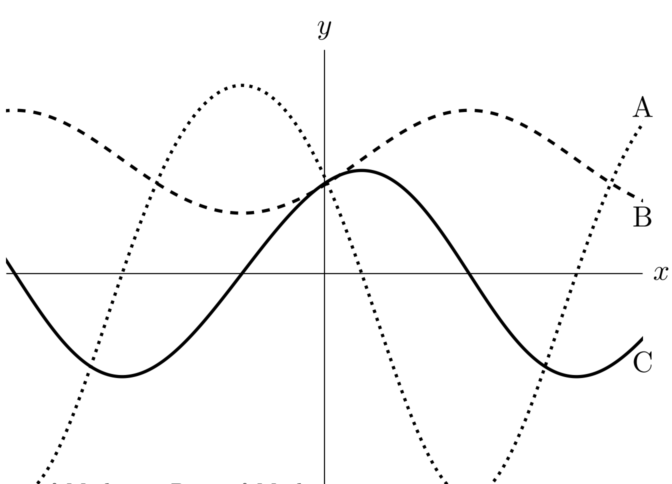
Solution: Since $g''(x) > 0$ on the domain of g, the graph of $g(x)$ is concave up near $x = 1$, so $L(x)$ underestimates $g(x)$ near $x = 1$.

ii. List the x-values of all inflection points of $f(x)$, or write NONE if $f(x)$ has no inflection points. Briefly justify your answer.

Solution: Since $g''(x) > 0$ on the domain of g, we have $f''(x) > 0$ for all $x < 1$. Similarly, since $h''(x) < 0$ on the domain of h, we have $f''(x) < 0$ for all $x > 1$. So $f(x)$ is concave up on $(-\infty, 1)$ and concave down on $(1, \infty)$, which means $f(x)$ has exactly one inflection point, namely at $x = 1$ where its concavity changes.

Answer: $x =$ 1

6. [4 points] Shown below are portions of the graphs of the functions $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. *No work or justification is needed.*



Answer: $f(x)$: B

$f'(x)$: C

$f''(x)$: A