

1. (7 pts) Let $q(x) = (1 + e^{-x})^{-1}$

a) (4 pts) Calculate the derivative $q'(x)$. No need to simplify.

$$\begin{aligned} q'(x) &= \frac{-(1 + e^{-x})'}{(1 + e^{-x})^2} && \text{(power law)} \\ &= \frac{-(-e^{-x})}{(1 + e^{-x})^2} && \text{(exponential / chain rule)} \\ &= e^{-x} / (1 + e^{-x})^2 \end{aligned}$$

b) (3 pts) Imagine inserting your answer from part a) into the integral below. What does the fundamental theorem of calculus tell you about the integral? You need not calculate a value.

$$\begin{aligned} \int_{-1000}^{1000} (\text{answer from part a}) dx &= \int_{-1000}^{1000} q'(x) dx \\ &= q(1000) - q(-1000) \\ \text{or,} &= (1 + e^{-1000})^{-1} - (1 + e^{1000})^{-1} \end{aligned}$$

2. (5 pts) $\int_3^9 (5 - 2f(x)) dx = 22$. Find $\int_3^9 f(x) dx$.

$$\begin{aligned} 22 &= \int_3^9 (5 - 2f(x)) dx = \int_3^9 5 dx - 2 \int_3^9 f(x) dx \\ &= 30 - 2 \int_3^9 f(x) dx \end{aligned}$$

$$\text{Solving, } 2 \int_3^9 f(x) dx = 30 - 22 = 8$$

$$\int_3^9 f(x) dx = 4$$