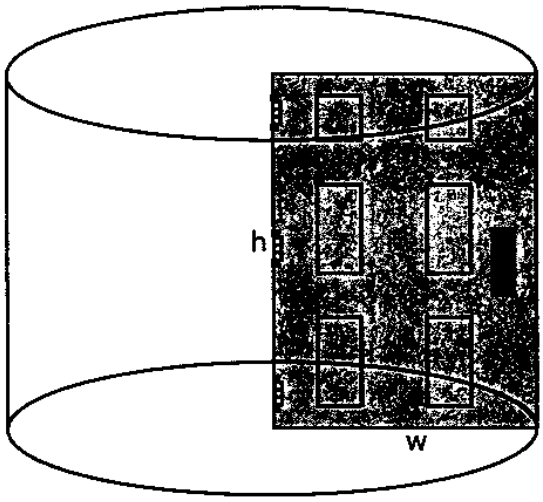
**10.** (10 pts) Your "Superfoamy Supercomfy Mattress" was a big hit! You've been promoted to Manager of the *Impractical Doors* subdivision of All Things Foam. Your latest brainchild is "The Great Transcen-Door," a door which rotates 360 degrees around its hinge. (See diagram). Architectural constraints force the volume of the cylinder in which the door rotates to be 3000 cubic feet. You believe that the door will be most pleasing if it has small perimeter. How can you design the (rectangular) door so that the perimeter of this rectangle is as small as possible? What is that perimeter? (The volume of a cylinder is $\pi r^2 h$, where $r$ is the base radius and $h$ the height. The door may indeed be impractical.)




We are given $\pi h w^2 = 3000$ cu. ft.

The perimeter of the door is $2h + 2w$.

Write $h = \frac{3000}{\pi w^2}$ (from the volume eqn.),

and substitute into the formula for the perimeter to get $P(w) = 2w + \frac{6000}{\pi w^2}$.

To minimize P, set $P' = 0 = 2 - \frac{2 \cdot 6000}{\pi w^3}$.

Solving gives $\pi w^3 = 6000$, or

$w = \sqrt[3]{\frac{6000}{\pi}}$, $h = \sqrt[3]{\frac{3000}{4\pi}}$. (Units = feet)

Or, $w = 12.4$ ft., $h = 6.2$ ft., $P = 37.2$ ft.