

7. (6 pts) In this problem we will investigate the family of functions

$$f(x) = a \ln(x) - bx.$$

Calculate values of a and b which cause such a function to have a critical point at $(2, 1)$.

For the graph of $f(x)$ to pass through the point $(2, 1)$ we need:

$$\textcircled{1} \quad a \ln(2) - b \cdot 2 = 1$$

For $(2, 1)$ to be a critical point, we need $f'(2) = 0$, that is, since

$$f'(x) = \frac{a}{x} - b, \quad \text{we want}$$

$$\textcircled{2} \quad f'(2) = 0 = \frac{a}{2} - b.$$

From $\textcircled{2}$, we get $b = \frac{a}{2}$. Substitute this into $\textcircled{1}$ to get

$$\begin{aligned} 1 &= a \ln 2 - \frac{a}{2} \cdot 2 = a \ln(2) - a \\ &= a (\ln(2) - 1) \end{aligned}$$

$$\text{Solving,} \quad a = \frac{1}{\ln(2) - 1}, \quad b = \frac{1}{2(\ln(2) - 1)}$$