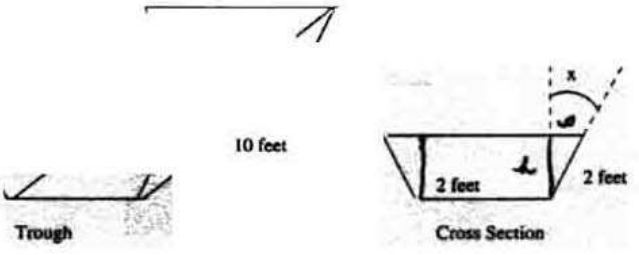
10. (10 points) A trough, as shown in the figure, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long and are to be placed so that they make an angle x with the vertical.



$\frac{h}{2} = \cos x$ $\quad h = 2\cos x$

$\frac{w}{2} = \sin x$ $\quad w = 2\sin x$

(a) What is the area, in terms of x, of a cross section of the trough perpendicular its long side? What is the volume of the trough? Show your work.

$$A(x) = 2(2\cos x) + 2\left(\tfrac{1}{2}(2\sin x)(2\cos x)\right) = 4\cos x + 4\cos x \sin x \text{ ft}^2$$

$$V(x) = 10(4\cos x + 4\cos x \sin x) \text{ ft}^3$$

(b) What angle x will give the trough of largest volume, and what is that volume? Explain how you found your answer, along with any supporting evidence (you may use your calculator).

$$V'(x) = 40(-\sin x + \cos x(\cos x) - \sin x \sin x)$$

$$= 40(-\sin x + \cos^2 x - \sin^2 x)$$

$$= 40(-\sin x + 1 - \sin^2 x - \sin^2 x)$$

$$40(1 - \sin x - 2\sin^2 x)$$

$V' = 0$ if $(1 - 2\sin x)(1 + \sin x) = 0$

$\rightarrow \sin x = \frac{1}{2}$ or ~~$\sin x = -1$~~ can be excluded

It makes sense to restrict x, $0 \le x \le \frac{\pi}{2}$, so CP on that interval is @ $x = \frac{\pi}{6}$. $V(0) = 40$, $V(\frac{\pi}{2}) = 0$, $V(\frac{\pi}{6}) \approx 51.96$

So max occurs when $x = \pi/6$ & max volume is $\approx 52 \text{ ft}^3$.