4. (15 points) **(a)** Given that $g(x)$ $f(e^{-x})$, where f is a function with $f'(1) = 3$ and $f''(1) = -5$, compute $g'(0)$ and $g''(0)$.

ANSWERS: $g'(0) =$ -3 $\qquad g''(0) =$ -2

$$g(x) = f(e^{-x});\quad g'(x) = f'(e^{-x})\cdot e^{-x}(-1)$$
$$g'(0) = f'(1)(1)(-1) = 3(-1) = -3$$
$$g''(x) = f''(e^{-x})(-e^{-x})(e^{-x})(-1) + f'(e^{-x})e^{-x}$$
$$g''(0) = f''(1)(-1)(1)(-1) + (3)(1) = -5 + 3 = -2$$

(b) Show that the point $x =$ $y = \pi/4$ lies on the curve

$$2 + xy = \frac{\pi}{4} + x^2 + \tan(y) \longrightarrow 2 + \frac{\pi}{4} = \frac{\pi}{4} + + = \frac{\pi}{4} + 2 \checkmark$$

and calculate dy/dx at this point

$$x\frac{dy}{dx} = 2x + \frac{1}{\cos^2 y}\frac{dy}{dx}$$

when $x = 1$, $y = \frac{\pi}{4}$

$$\frac{\pi}{4} + \frac{dy}{dx} = 2 + \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}\frac{dy}{dx} \longrightarrow \frac{dy}{dx}(2-1) = \frac{\pi}{4} - 2$$

$$\frac{dy}{dx} = \frac{\pi}{4} - 2$$

(c) The cost function $C(q)$ represents the cost in dollars of producting q units of some good and the revenue function $R(q)$ represents the revenue in dollars received by selling q units of the good. If $C'(500) = 100$ and $R'(500) = 125$, should the quantity produced be increased or decreased from $q = 500$ in order to increase profits? Explain the reason for your answer.

Inc

$$\pi(q) = R(q) - C(q)$$
$$\pi'(q) = R'(q) - C'(q)$$
$$\pi'(500) = 125 - 100 = 25$$

Profit is increasing @ $q = 500$

$\pi'(500) = 25$ essentially means an additional unit will bring in an extra \$25 in profit