



6. (7 points) The graph shown above is of the function  $f(x) = \ln(3x)$ .

(a) Without computing a final answer (i.e. do not use your calculator), give an exact expression for the left hand sum using four subdivisions that estimates the integral of  $f$  on the interval  $1 \leq x \leq 9$ .

$$LHS_{(4)} = (\ln 3 + \ln 9 + \ln 15 + \ln 21)(2)$$

$$\Delta x = \frac{9-1}{4} = 2$$

(b) Illustrate the sum on the graph

(c) Would your sum be an underestimate or an overestimate of  $\int_1^9 \ln(3x) dx$ ? Explain why.

The LHS is an underestimate because the function is increasing on  $[1, 9]$ .