$f(x)>0 \quad f'(x)<0 \quad f''(x)<0$ 7

7. (9 points) Let f be a function that is positive, decreasing, and concave down for $1 < x < 2$. Let g be the function defined by $g(x) = 1/f(x)$. Use the methods of calculus to show why the statements in **(a)** and **(b)** are true.

(a) g is increasing for $1 < x < 2$.

$g(x) = \frac{1}{f(x)} \longrightarrow g'(x) = \frac{-f'(x)}{(f(x))^2} \longrightarrow \frac{--}{+} = \frac{+}{+}$

(b) g is concave up for $1 < x < 2$.

$$g''(x) = \frac{(f(x))^2(-f''(x)) + f'(x)\,2(f(x))f'(x)}{(f(x))^4}$$

$$\longrightarrow \frac{(+)(-)(-) + (-)(+)(-)}{+} = \frac{+++}{+} = +$$

$f>0 \quad f'<0 \quad f''>0$

(c) If f is positive, decreasing, and *concave up* for $1 < x < 2$, is $g(x) = 1/f(x)$ always increasing and concave down for $1 < x < 2$? Explain why or why not.

$g'(x) = \frac{f'(x)}{(f(x))^2} \longrightarrow \frac{--}{+} = \frac{+}{+}$ → still increasing

$g''(x) \Rightarrow + \frac{(-)(+) + (-)(+)(-)}{+} \longrightarrow \frac{(-) + (+)}{+}$

Cannot say for sure where g is conc. up or concave down.