(11.) (5 points) Suppose that on your visit home over break you meet a friend who is now taking precalculus at your old high school. He knows the formula "distance travelled = rate × time." He also knows some students who are taking the calculus course at the high school, and he has heard there is a more general formula, "distance travelled = area under the velocity curve," that computes the distance, even when the velocity is not constant. He asked those students to explain this second formula, but they just shrugged and said he would have to wait until he learned calculus to get an explanation.

Write down what you would tell your friend to explain why the second formula holds and how it is related to the formula he has learned in precalculus. Be sure to include any appropriately labelled graphs you might draw in making your explanation.

The precalculus formula of $d = v\,t$ gives the formula relating the distance, time and velocity when the velocity is constant over time. One can think of this as the area under a velocity curve given by a horizontal line, as can be seen in (Fig. 1). However, when the velocity function varies

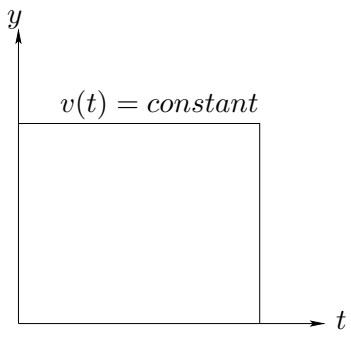

*Fig.*1

with time, we can approximate it on small intervals to be a constant function. The height of the rectangle gives an approximation to the actual velocity function. Therefore the actual distance travelled over this small interval is approximately the distance travelled by the constant velocity we are approximating the actual velocity by. So the actual distance is approximately the height of the rectangle × the time interval we are using. Note that this is actually the area of the rectangle! See (Fig. 2).

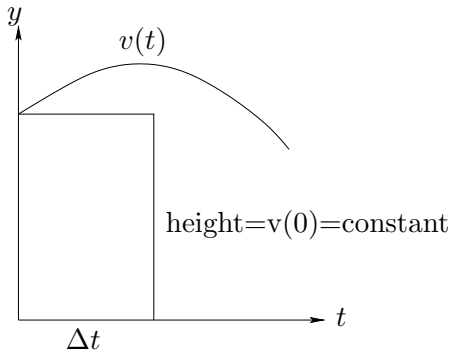

*Fig.*2

Now to get an approximation of the total distance travelled, one merely adds up the various rectangles used in the approximation. As the time interval used decreases to 0, the number of rectangles used increases to infinity and we see that the area of the rectangles actually fills out the entire area under the velocity curve. See (Fig. 3) for the cases of 3 rectangles and 6 rectangles. This is how one goes from the simple precalculus formula of distance = rate × time to the area

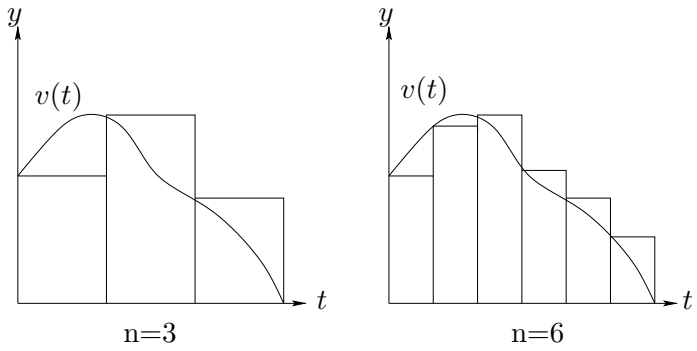

*Fig.*3

under the velocity curve being the distance travelled.

Please **print** your name here: Name _____

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