

- (5.) (5 points) Let $f(x) = 1/x$. Use the *limit definition* of the derivative (and some algebra) to compute $f'(x)$. [Show **all** work.]

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x^2 + xh}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-h}{x^2 + xh}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{x^2 + xh} \\
 &= \frac{-1}{x^2}
 \end{aligned}$$

- (6.) (8 points)

- (a) Given $F(x) = x \ln(x) - x + C$, show that $F'(x) = \ln(x)$. [Show all your work.]

$$\begin{aligned}
 F'(x) &= \ln(x) + x \cdot \frac{1}{x} - 1 + 0 \\
 &= \ln(x) + 1 - 1 \\
 &= \ln(x)
 \end{aligned}$$

- (b) If $F(1) = 3$, find C .

$$\begin{aligned}
 F(1) &= 1 \cdot \ln(1) - 1 + C \\
 &= 1 \cdot 0 - 1 + C \\
 &= -1 + C \\
 &= 3,
 \end{aligned}$$

so $C = 4$.

- (c) Evaluate $\int_1^3 \ln(x) dx$. [Give an *exact* answer, not an approximation.]

Since $F'(x) = \ln(x)$, the fundamental theorem of calculus says that:

$$\begin{aligned}
 \int_1^3 \ln(x) dx &= F(3) - F(1) \\
 &= (3 \cdot \ln(3) - 3 + C) - (1 \cdot \ln(1) - 1 + C) \\
 &= 3 \cdot \ln(3) - 2.
 \end{aligned}$$