(5.) (5 points) Let $f(x) = 1/x$. Use the *limit definition* of the derivative (and some algebra) to compute $f'(x)$. [Show **all** work.]

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h\to 0} \frac{\frac{x-(x+h)}{x^2+xh}}{h} \\ &= \lim_{h\to 0} \frac{\frac{-h}{x^2+xh}}{h} \\ &= \lim_{h\to 0} \frac{-1}{x^2+xh} \\ &= \frac{-1}{x^2} \end{aligned}$$

(6.) (8 points)

(a) Given $F(x) = x\ln(x) - x + C$, show that $F'(x) = \ln(x)$. [Show all your work.]

$$\begin{aligned} F'(x) &= \ln(x) + x \cdot \frac{1}{x} - 1 + 0 \\ &= \ln(x) + 1 - 1 \\ &= \ln(x) \end{aligned}$$

(b) If $F(1) = 3$, find C.

$$\begin{aligned} F(1) &= 1 \cdot \ln(1) - 1 + C \\ &= 1 \cdot 0 - 1 + C \\ &= -1 + C \\ &= 3, \end{aligned}$$

so $C = 4$.

(c) Evaluate $\int_1^3 \ln(x)\,dx$. [Give and *exact* answer, not an approximation.]

Since $F'(x) = \ln(x)$, the fundamental theorem of calculus says that:

$$\begin{aligned} \int_1^3 \ln(x)\,dx &= F(3) - F(1) \\ &= (3 \cdot \ln(3) - 3 + C) - (1 \cdot \ln(1) - 1 + C) \\ &= 3 \cdot \ln(3) - 2. \end{aligned}$$