(7.) (8 points) The following is a table of values of a continuous function f:

x	0	20	40	60	80	100
$f(x)$	1.2	2.8	4.0	4.7	5.1	5.2

(a) Use a left-hand sum with five intervals to estimate the definite integral $\int_0^{100} f(x)\,dx$. Show your work.

We use five rectangles to estimate the area. Each rectangle has width 20, and height given by the value of the function at the left endpoint. Thus we have:

$$\int_0^{100} f(x)\,dx \approx 20 \cdot (1.2 + 2.8 + 4.0 + 4.7 + 5.1) = 356.$$

(b) Assuming that f is monotonic (i.e., always increasing or decreasing on the interval), how many intervals must you use to guarantee that the left hand sum is within .1 of the actual value of the integral?

Since f is monotonic, the actual value of the integral is between the value of any left hand sum and any right hand sum. If we use n intervals for both the left hand sum and right hand sum, then we have the formula:

$$\begin{aligned} RHS - LHS &= (100/n) \cdot (5.2 - 1.2) \\ &= (400/n). \end{aligned}$$

If we can get this value to be less than .1, then we will guarantee that the left hand sum is within .1 of the true value of the integral. So we need:

$$\begin{aligned} (400/n) &< .1 \\ 400 &< .1n \\ 4000 &< n. \end{aligned}$$

(c) Given the information you have, is your left-hand sum an underestimate or an overestimate? Explain.

Because f is monotonically increasing, the value of any left-hand sum will be an underestimate of the actual value of the integral.