

- (9.) (8 points) Winter is here! Soon we will have icicles. Consider an icicle in the shape of a right circular cone. The sun is causing the icicle to lengthen. As its length, h , is increasing at the rate of 0.5 cm/hr, the radius, r , of the cone is decreasing at the rate of 0.02 cm/hr. When the icicle is 12 cm long and its radius is 1 cm, is the volume of the icicle increasing or decreasing? At what rate is the volume changing? [The volume of a right circular cone is given by $V = \frac{1}{3}\pi r^2 h$. Note that in this problem, both h and r are functions of time.]

Take the derivative of V with respect to time:

$$\begin{aligned}\frac{dV}{dt} &= \frac{\pi}{3} \left(2r \cdot \frac{dr}{dt} \cdot h + r^2 \cdot \frac{dh}{dt} \right) \\ &= \frac{\pi}{3} (2 \cdot 1 \cdot (-.02) \cdot 12 + 1^2 \cdot .5) \\ &= \frac{\pi}{3} (-.48 + .5) \\ &= \frac{\pi}{3} \cdot (.02) \approx 0.0209 \text{ cm}^3/\text{hr}.\end{aligned}$$

This number is positive, so the volume is increasing at the rate of approximately 0.02 cm³/hr.