10. (13 points) Alas, you returned home without a cent to your name–but you had a great time! You are hitching a ride back to school with friends. You have agreed to take a small suitcase in order to save space, but you realize that you also need to take all the souvenirs that you are bringing back to your friends at school. You determine that you need 3000 in^3 of space for those gifts. Your friend agrees to strap a box to the roof of his car. You are worried about the straps crushing the box and about the weather, so you decide to have a box made. The box-making place agrees to make a rectangular box with reinforced sides and weatherproofing on the top and sides. The sides come in a standard height of 6 inches and cost \$0.25 per in^2. The top costs \$0.15 per in^2, and the materials for the bottom cost \$0.05 per in^2. There is also a \$10 charge for labor.

(a) You are going to have to go into debt to get this box, so find the dimensions of the box with the standard height sides (6 inches) that will minimize the cost.

We begin with the first condition. If the box has dimensions $6 \times l \times w$, then the volume of the box is given by $V = 6lw = 3000$. Solving for w we have $w = \frac{500}{l}$. Next we need to find the cost formula. Using the prices given in the problem and what we have for w we get:

$$\begin{aligned} C &= .15lw + .25[12w + 12l] + 0.05lw \\ &= .15 \cdot 500 + .25\left[\frac{12 \cdot 500}{l} + 12l\right] + 0.05 \cdot 500 + 10. \end{aligned}$$

To minimize this we take the derivative with respect to l and set it equal to zero:

$$.25\left[-\frac{6000}{l^2} + 12\right] = 0.$$

Using that $l > 0$ we solve this to get $l = 22.36$ inches. Thus $w = \frac{500}{22.36} = 22.36$ inches. The physical situation easily shows us that the endpoints will not work as if $l = 0$ or $w = 0$ then we cannot satisfy the volume condition.

(b) How much is the box going to cost?

We merely need to plug $l = w = 22.36$ inches into the formula for cost:

$$C(22.36) = \$244.16.$$