**9.** (4+3+3 points) After watching hummingbirds for a while, you head to the beach to tan. Shortly after you begin tanning you are overcome with boredom. You decide to pass your time by digging a glorious hole in the ground. Just as a satisfactory hole begins to emerge from your efforts, the tide comes in and waves begin to wash sand back into your hole. You continue digging, but your enthusiasm is ending. Finally, you realize you're defeated and give up. The rate of sand flow into the hole due to the incoming waves and the rate of flow of sand out of the hole due to your digging are graphed below. Assume time $t = 0$ corresponds to the first wave coming in.
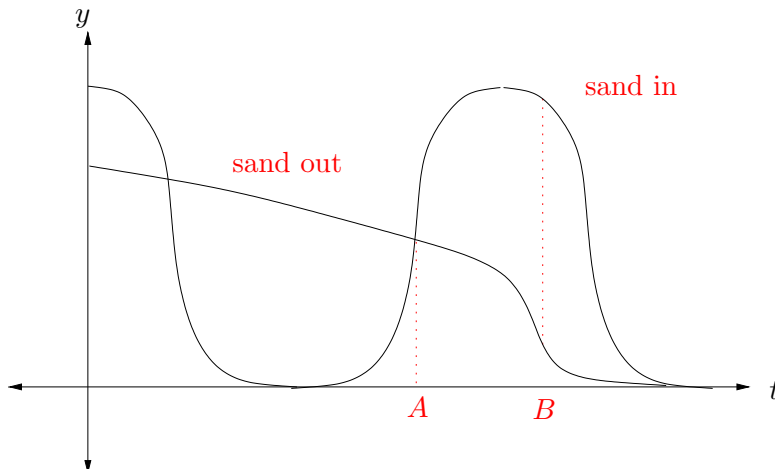
(a) Label which curve corresponds to the rate of sand going out of the hole and which curve corresponds to the rate of sand being washed into the hole by the waves. Explain how your arrived at your answer.

As the waves begin to wash in you tire and eventually give up. This should correspond to a rate out of the hole that is slowly decreasing over time and eventually drops to zero.
Waves wash sand into the hole at a high rate as they wash in, then the rate drops to zero as the wave recedes. This pattern repeats with each wave. This is why the graphs are labelled as they are.

(b) Mark and label the point on the graph when the least amount of sand was in the hole. Label the point as A. Explain how you arrived at your answer.

The points to consider here is $t = 0$ as the first wave washes in and the point marked A on the graph above. After point A more sand is going into the hole then you are shovelling out, so clearly the hole is filling at this point. One sees that the minimum amount of sand must be in the hole at point A because the volume between rate out and rate in between 0 and A is positive, so you have removed more sand over this period then the waves have washed in.

(c) Mark and label the point on the graph when the amount of sand in the hole was increasing most rapidly? Label the point as B. Explain how you arrived at your answer.

The amount of sand is increasing most rapidly when the difference between sand in - sand out is the largest.