

4. (8 points) Consider the function $f(x) = e^{-x^2}$.

(a) Use a right-hand sum with four equal subdivisions to estimate $\int_0^2 f(x)dx$.

With four subdivisions we must use $\Delta x = \frac{(2-0)}{4} = 0.5$. The right-hand sum estimate is therefore,

$$0.5(f(0.5) + f(1) + f(1.5) + f(2)) = 0.6352$$

(b) Without computing the integral from part (a), determine whether your estimate is an overestimate or an underestimate. Justify your answer.

Note that $f'(x) = -2xe^{-x^2}$. Since $e^{-x^2} > 0$ for all $x > 0$ and $-2x < 0$ for all $x > 0$, we see that $f'(x) < 0$ for all $x > 0$. Therefore, $f(x)$ is decreasing for all $x > 0$ and so the right-hand sum estimate is an underestimate.