5. (14 points) Suppose that the temperature (in degrees Celsius) on December 10th at the North Pole was described by the function $f(t) = -0.3t^2 + 7t - 38$, where t is hours after midnight for values of $0 \leq t \leq 24$.

(a) Find the average rate of change in temperature between 5 am and 2 pm. Show your work.

By definition, the average rate of change of $f(t)$ over the interval $5 \leq t \leq 14$ is

$$\frac{\Delta f}{\Delta t} = \frac{f(14) - f(5)}{14 - 5} = 1.3 \text{ degrees per hour}$$

(b) Find the average temperature between the hours of 5 am and 2 pm. Show your work.

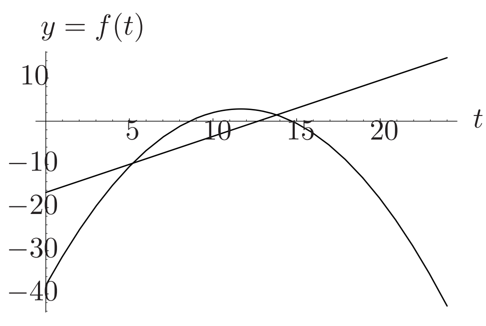
By definition, the average value of a function, f, on the interval $5 \leq t \leq 14$ is

$$\frac{1}{14-5}\int_5^{14} f(t)dt$$

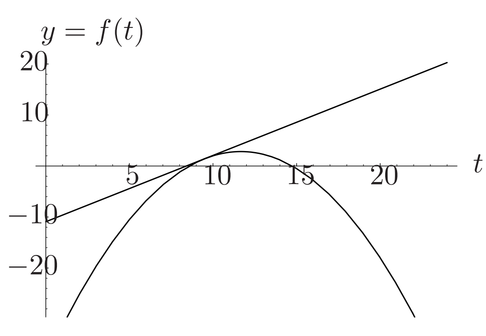
Note that $F(t) = -0.1t^3 + 3.5t^2 - 38t$ is an antiderivative for $f(t)$. Therefore, the average temperature is

$$\frac{F(14) - F(5)}{9} = -0.6 \text{ degrees}$$

(c and d) On the first sketch of $f(t)$ given in the figure below, show how the value from part (a) can be represented graphically. Use the second graph below, to approximate a time t for which $f'(t)$ is equal to the average rate of change of temperature from part (a). Show how this can be represented graphically. Carefully label and explain what you are indicating on each graph.



The value from part (a) can be represented as the **slope** of the line that joins the points $(5, -10)$ and $(14, 1.2)$. This line is shown in the figure above.



If f' is equal to the value from part (a), this can be represented graphically as the **slope** of a line tangent to the curve which has a slope of 1.3. This line is given in the graph above. The t value can be approximated on the graph.