8.(10 points) On Christmas Eve, the Grinch and Santa each head **first** for Joe's house. The Grinch usually likes to arrive at houses after Santa, but for this first stop the Grinch wants to get to the cookies before Santa can. (The cookies at Joe's are *exceptionally* good.) Assume that Santa is directly North of the house (therefore traveling due South) while the Grinch is directly East of the house (traveling due West–also flying, so as to try to get ahead of Santa). Assume that both Santa and the Grinch are flying at the same altitude.

Santa is moving at 30 miles per hour, and the Grinch is going 28 miles per hour. How fast is the distance between them changing when Santa is 120 miles from Joe's house and the Grinch is 160 miles from the house?

We need to relate the distance, D, between Santa and the Grinch to the distance, x, between Santa and Joe's house and the distance, y, between the Grinch and Joe's house. The fact is that x, y and D form the legs of a right triangle, where D is the hypotenus, so

$$D = \sqrt{x^2 + y^2} = (x^2 + y^2)^{1/2}.$$

Now, x, y and D are all changing with time, t, so by the chain rule

$$(*)\quad \frac{dD}{dt} = \frac{1}{2}(x^2 + y^2)^{-1/2}(2x\frac{dx}{dt} + 2y\frac{dy}{dt}).$$

Note that $\frac{dx}{dt} = -30$ (negative because Santa is moving towards Joe's house, so the distance between Santa and Joe's house is decreasing) and similarly, $\frac{dy}{dt} = -28$. When we plug $x = 120, y = 160, \frac{dx}{dt} = -30$, and $\frac{dy}{dt} = -28$ into $(*)$ we find that

$$\frac{dD}{dt} = -40.4$$

So the distance between Santa and the Grinch is decreasing at 40.4 miles per hour.