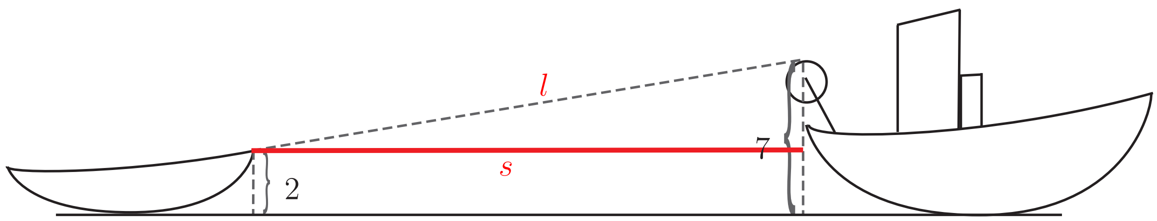
5. (10 points) A small boat has run out of gas. A cable is attached to the front of the boat 2 meters above the water. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of a tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below–not drawn to scale.]



(a) At what rate is the length of the cable between the two boats changing?

Let n represents the number of rotations of the wheel and l the length of the cable between the two boats. We are only interested in the rate at which l is changing, and l decreases by $2\pi(.5) = \pi$ meters per rotation. Since the wheel is being turned at 1 revolution per second then l is decreasing by π meters per second. In other words $dl/dt = -\pi \frac{\text{meters}}{\text{second}}$.

(b) How fast is the small boat being pulled forward when it is 10 meters away from the tugboat?

Let s represent the distance between the two boats (see the diagram). Using the Pythagorean Theorem, $s^2 + 25 = l^2$. Solving for s,

$$s = \sqrt{l^2 - 25}.$$

There is no ambiguity here because s and l are both positive distances. Differentiating this relation with respect to t gives:

$$\frac{ds}{dt} = \frac{l}{\sqrt{l^2 - 25}} \frac{dl}{dt}.$$

From the information in part (a) we know that $dl/dt = -\pi \frac{\text{m}}{\text{s}}$. Additionally we want to know ds/dt when $s = 10$ meters, using again that $l^2 = s^2 + 25$ we see that $l = 5\sqrt{5}$ when $s = 10$. Consequently,

$$\frac{ds}{dt} = \frac{5\sqrt{5}}{\sqrt{(5\sqrt{5})^2 - 25}}(-\pi) = -\frac{\sqrt{5}}{2}\pi \frac{\text{m}}{\text{s}}.$$