6. Suppose $H(c)$ gives the average temperature, in degrees, that can be maintained in Oscar's apartment during the month of December as a function of the cost of the heating bill, c, in dollars. In complete sentences, give a practical interpretation of the following:

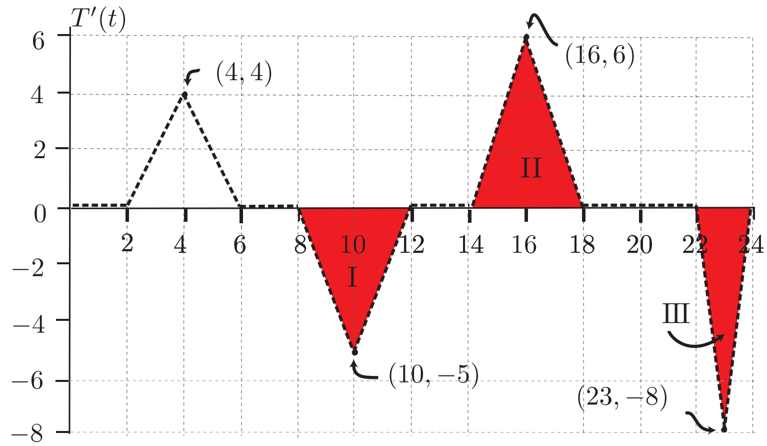
   (a) (3 points) $H(50) = 65$

   The practical interpretation of $H(50) = 65$ is that if Oscar's heating bill costs \$50.00 in December then he is able to maintain an average temperature during that month of 65°.

   (b) (3 points) $H'(50) = 2$

   The practical interpretation of $H'(50) = 2$ is that if Oscar's December heating bill increases from \$50.00 to \$51.00, then the average temperature he can maintain during that month will change from 65° to approximately 67°.

Suppose $T(t)$ gives the temperature in Oscar's apartment on December 18th in °F as a function of the time, t, in hours since 12:00 midnight. Below is a graph of $T'(t)$: (**NOTE: the graph is of $T'(t)$.**)



   (c) (6 points) When Oscar gets home from work at 6 pm the temperature in his apartment is 67 degrees. What was the temperature when he left for work at 8 am?

   Using the Fundamental Theorem of Calculus we know that

$$\int_8^{18} T'(t)dt = T(18) - T(8).$$

   We are given that $T(18) = 67°$. Computing the areas I and II on the graph of T' we know that the $\int_8^{18} T'(t)dt$ =-II+I= $-10 + 12 = 2$ Consequently, $2 = 67° - T(8)$. Thus the temperature at 8 am is $T(8) = 65°$.

   (d) (4 points) If the temperature at 6 pm is 67 degrees, what is the minimum temperature in the apartment on December 18th?

   When $T'(t) > 0$ ($T'(t) < 0$) then T is increasing (decreasing) and when $T'(x)$ is 0 on an interval the temperature is constant. Thus for $12 < t < 2$, $6 < t < 8$, $12 < t < 14$ and $18 < t < 22$ the temperature is not changing, for $2 < t < 6$ and $14 < t < 18$ the temperature is increasing, while it is decreasing for $8 < t < 12$ and $22 < t < 24$. Consequently the minimum temperature either occurs at $t = 0, t = 12$ or $t = 24$. Using the Fundamental Theorem of Calculus, and our knowledge of interpreting integrals as areas we have, $T(12) = T(18) - \int_{12}^{18} T'(t)dt = 67 - II = 67 - 12 = 55°$ and $T(24) = T(18) + \int_{18}^{24} T'(t)dt = 67 - III = 67 - 8 = 59°$, and similarly $T(0) = 59°$. Consequently, the minimum temperature occurs anywhere between noon and 2pm. This minimum temperature is 55°F.