5. (10 points) Madam Whippy's ice-cream store has a vending machine that pumps out manilla vanilla at a constant rate of 2 cm^3 per second. If you are collecting the ice-cream in a cone of maximum radius 5 cm and height 10 cm, how fast is the radius of the surface of the ice-cream changing when the height of ice cream in the cone is 6 cm? Assume that the ice cream is soft-serve and fills the cone with a flat surface.

[You may need: Volume of a cone of height h and radius r is given as $V = \frac{1}{3}\pi r^2 h$.]
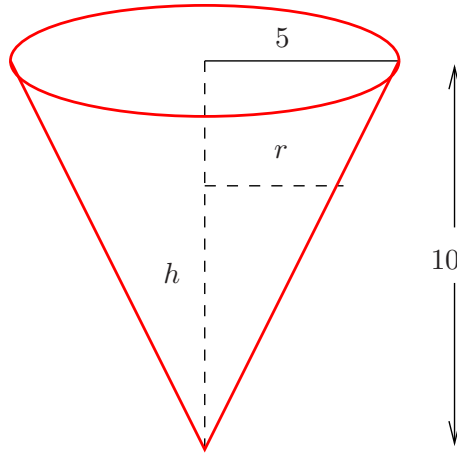
Using similar triangles,

$$\frac{r}{h} = \frac{5}{10} \rightarrow h = 2r$$

Thus $V = \frac{2}{3}\pi r^3$ and

$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$

and since we are given $\frac{dV}{dt} = 2$, we have

$$2 = 2\pi r^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{1}{\pi r^2}$$

which at $h = 6$ and $r = 3$ gives

$$\frac{dr}{dt} = \frac{1}{9\pi} \quad \text{cm/s.}$$