7. (16 points) One winter a huge storm hit the Ann Arbor area hours earlier than expected. Snow began falling at midnight and quickly began to accumulate. Although emergency conditions were put into place immediately, snow removal teams were unable to get onto the roads until 2:00 a.m. Particular attention was given along I-94, which needed to be cleared for heavy commuter traffic. The rate of snowfall along that highway strip is given by the graph below, where r(t) is in inches per hour, and t is hours past midnight. Assume that once the removal starts, plows can remove the snow at an average rate of 2 in/hour. Use the graph to find or to estimate the following:



[Answers below are not unique.]

1

(a) The amount of snowfall that had accumulated prior to the time the plows got on the road at 2:00 a.m.

Note, each rectangle is 1/2 by 1 so each rectangle has area 1/2. Using rectangles to approximate the area under r from t = 0 to 2, gives approximately 9 rectangles= 1/2(9)=4.5 inches.

(b) The rate at which the snow level is changing at 3:00 a.m.

At 3:00 a.m., the snow is falling at approximately 3.75 in/hour and is being plowed at 2 in/hour. Thus, it is is accumulating at approximately 1.75 inches per hour.

(c) The time when the level of accumulated snow is maximum.

The level will be at the maximum when the plowing rate overtakes the falling rate–or at approximaely 5:45 a.m.

(d) The time when the highway was cleared of snow, assuming that the snowfall stopped at 8:00 a.m.

We are interested in when the area under r is cleared–which is equivalent to considering how long it would take to clear the snowfall represented by the shaded area in the figure above. We have already found the area from t = 0 to t = 2 to represent approximately 4.5 inches, and there are approximately 13 rectangles in the shaded regions between t = 2 and t = 8 when the snow stops. Thus, we need to clear 4.5 + 0.5(13) = 11 inches of snow in excess of what has been cleared by the plows at 5:45 a.m. At a rate of 2 inches per hour, it will take approximately 5.5 hours after the plowing rate exceeds the falling rate– or until around 11:15 a.m.