1. [12 points] In the 17th century, a ship’s navigator would estimate the distance the ship has traveled using readings of the ship’s velocity, \(v(t)\), in knots (nautical miles per hour). Suppose that between noon and 3:00 pm a certain galleon is traveling with the wind and against the ocean current, and that its velocity is given as the difference between the wind velocity \(w(t)\) and the velocity of the ocean current \(c(t)\), so that \(v(t) = w(t) - c(t)\), where \(t\) is in hours since noon. Consider the wind and ocean velocities for various times between noon and 3:00 p.m., given by the graphs below:

\[
\begin{align*}
\text{\(w(t)\)} & \quad \text{\(c(t)\)} \\
t_1 & \quad t_2 & \quad t_3 & \quad t_4 & \quad t_5 & \quad t_6 & \quad t_7
\end{align*}
\]

a. [1 point] Using integral notation write an expression giving the distance the ship traveled from noon to 3:00 pm. Give units.

Solution: \(d = \int_0^3 v(t)\,dt\), with the distance in nautical miles.

b. [1 point] Using integral notation write an expression giving the average velocity of the ship between noon and 3:00 pm. Give units.

Solution: \(v_{av} = \frac{1}{3} \int_0^3 v(t)\,dt\), with the distance in nautical miles/hour, or knots.

c. [2 points] For what intervals was the ship’s velocity positive?

Solution: The ship’s velocity is positive when \(w(t) > c(t)\), which happens on the intervals \((t_1, t_3)\) and \((t_5, t_7)\).

d. [2 points] For what \(t\) values was the ship not moving towards its destination?

Solution: Since this happens when the ship’s velocity is zero or negative, the \(t\) values are \([t_0, t_1], [t_3, t_5]\) and \(t_7\).

e. [2 points] For what intervals was the ship’s velocity increasing?

Solution: The ship’s velocity is increasing when the acceleration is positive, and since \(a(t) = v'(t) = w'(t) - c'(t)\), in order for \(a(t) > 0\) we need \(w'(t) > c'(t)\), i.e. that the slope of the tangent line to \(w(t)\) is greater than the slope of the tangent line to \(c(t)\). This happens on the intervals \((t_0, t_2)\) and \((t_4, t_6)\).

f. [4 points] Please circle each integral which is positive and underline each integral which is negative.

Solution:

\[
\begin{align*}
\int_{t_1}^{t_3} v(t)\,dt & \quad \int_{t_5}^{t_7} v(t)\,dt & \quad \int_{t_0}^{t_5} w(t)\,dt & \quad \int_{t_5}^{t_7} c(t)\,dt
\end{align*}
\]