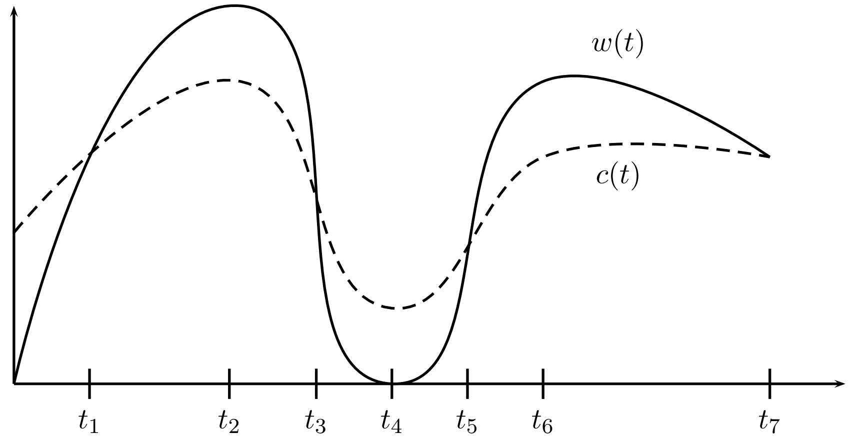
**1.** [12 points] In the 17th century, a ship's navigator would estimate the distance the ship has traveled using readings of the ship's velocity, $v(t)$, in knots (nautical miles per hour). Suppose that between noon and 3:00 pm a certain galleon is traveling with the wind and against the ocean current, and that its velocity is given as the difference between the wind velocity $w(t)$ and the velocity of the ocean current $c(t)$, so that $v(t) = w(t) - c(t)$, where $t$ is in hours since noon. Consider the wind and ocean velocities for various times between noon and 3:00 p.m., given by the graphs below:



**a.** [1 point] Using *integral notation* write an expression giving the distance the ship traveled from noon to 3:00 pm. Give units.

*Solution:* $d = \int_0^3 v(t)dt$, with the distance in nautical miles.

**b.** [1 point] Using *integral notation* write an expression giving the average velocity of the ship between noon and 3:00 pm. Give units.

*Solution:* $v_{av} = \frac{1}{3}\int_0^3 v(t)dt$, with the distance in nautical miles/hour, or knots.

**c.** [2 points] For what intervals was the ship's velocity positive?

*Solution:* The ship's velocity is positive when $w(t) > c(t)$, which happens on the intervals $(t_1, t_3)$ and $(t_5, t_7)$.

**d.** [2 points] For what $t$ values was the ship not moving towards its destination?

*Solution:* Since this happens when the ship's velocity is zero or negative, the $t$ values are $[t_0, t_1]$, $[t_3, t_5]$ and $t_7$.

**e.** [2 points] For what intervals was the ship's velocity increasing?

*Solution:* The ship's velocity is increasing when the acceleration is positive, and since $a(t) = v'(t) = w'(t) - c'(t)$, in order for $a(t) > 0$ we need $w'(t) > c'(t)$, *i.e.* that the slope of the tangent line to $w(t)$ is greater than the slope of the tangent line to $c(t)$. This happens on the intervals $(t_0, t_2)$ and $(t_4, t_6)$.

**f.** [4 points] Please circle each integral which is positive and underline each integral which is negative.

*Solution:* [✓ $\int_{t_1}^{t_3} v(t)dt$] [✓ $\int_{t_5}^{t_7} v(t)dt$] [✓ $\int_{t_0}^{t_7} w(t)dt$] [✓ $\int_{t_3}^{t_5} c(t)dt$]