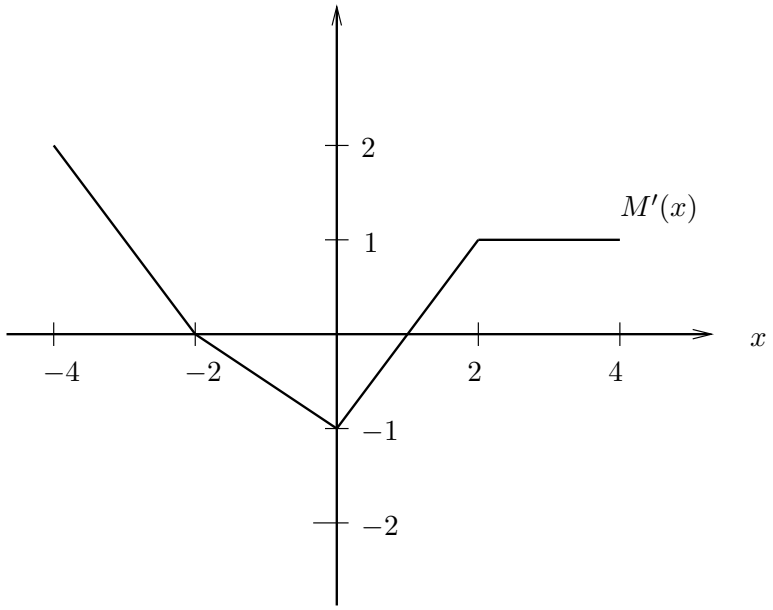
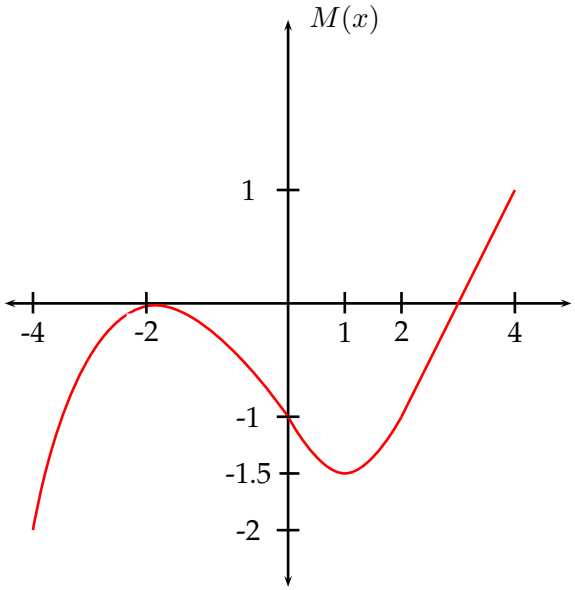
2. [10 points] The graph of the *derivative* of the continuous function $M(x)$ is given below.



Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$ on the axes below. Give the coordinates of all critical points, inflection points and endpoints of M on the interval [-4,4].

Solution:



To find the y-values, we have used $M(b) = M(-4) + \int_{-4}^{b} M'(x)dx = -2 + \int_{-4}^{b} M'(x)dx$, along with the formulas of the area of a triangle and rectangle to find the integrals. For example, $M(-2) = -2 + \int_{-4}^{-2} M'(x)dx = -2 + (1/2)(2)(2) = 0$. For $-4 < x < -2$, $M'(x)$ is positive and decreasing, so $M(x)$ is increasing and concave down. We can reason similarly to determine the shape of the graph of $M(x)$ over the intervals $-2 < x < 0, 0 < x < 2$ and $2 < x < 4$, giving critical points at $(-2, 0)$ and $(1, -1.5)$. The inflection point is at $(0, -1)$, and the endpoints are $(-4, -2)$ and $(4, 1)$.