3. [20 points] The following questions are each meant to have short computation times. Each question is worth 4 points.

a. [4 points] If $f(x)$ is even and $\int_{-2}^{2}(f(-x)-3)\,dx = 8$, find $\int_{0}^{2} f(x)\,dx$.

Solution: By definition, f being even means that $f(-x) = f(x)$. Now, $\int_{-2}^{2}(f(-x)-3)dx = \int_{-2}^{2} f(x)dx - 3\int_{-2}^{2} 1dx = \int_{-2}^{2} f(x)dx - 12$. Then, since $\int_{-2}^{2} f(x)dx = 2\int_{0}^{2} f(x)dx$, because f is even, we get that $2\int_{0}^{2} f(x)dx - 12 = 8$, which gives $\int_{0}^{2} f(x)dx = 10$.

b. [4 points] The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

Solution: We have that $\frac{1}{2-c}\int_{c}^{2}\frac{10}{x^2}dx = 5$. Since $\int_{c}^{2}\frac{10}{x^2}dx = 10\left(\frac{-1}{2}+\frac{1}{c}\right) = 10\left(\frac{1}{c}-\frac{1}{2}\right) = 10\left(\frac{2-c}{2c}\right) = 5\left(\frac{2-c}{c}\right)$, then the left hand side of our equation becomes $\frac{1}{2-c}\left(\frac{5(2-c)}{c}\right) = \frac{5}{c}$. Thus, solving $\frac{5}{c} = 5$ yields $c = 1$.

c. [4 points] If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_{3}^{27} R(t)\,dt$ means in this context.

Solution: The expression $\int_{3}^{27} R(t)dt$ represents the total number of tickets sold for UMAir Flight 123 from 3 pm on Dec. 15, 2008 to 3 pm on Dec. 16, 2008.

d. [4 points] Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$. [Note: this is an edit of the original problem. Grading on the final took into account the original wording.]

Solution: First note that $f^{-1}(N) = t$, so that 325 represents the number of students. Thus, $(f^{-1})'(325) = 2$ means that when 325 students have turned in their exam, it will be approximately 2 minutes before the next student turns in an exam.

e. [4 points] Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2+1, & x \le 1 \\ 6\sin(\pi(x-0.5))+k, & x > 1 \end{cases}$$

Solution: Each function that makes up $h(x)$ is continuous on for all real numbers. Thus, the only place where we need to worry about $h(x)$ not being continuous is at $x = 1$. We know that $h(1) = (1)^2 + 1 = 2$, and that $\lim_{x\to 1^+} h(x) = 6\sin(\pi(1-0.5)) + k = 6 + k$. So, in order for $h(x)$ to be continuous at $x = 1$, we need that $\lim_{x\to 1^+} h(x) = h(1)$, or that $6 + k = 2$, which gives $k = -4$.