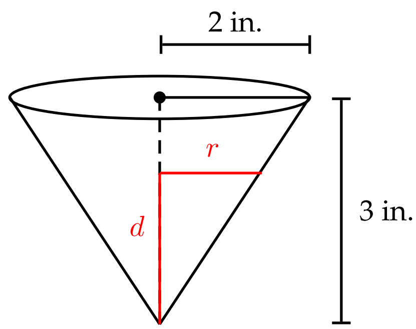
5. [15 points] Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter (see below). Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?
[Note: if d is depth of the water in the cone and the radius is r, the volume is given by $V = \frac{1}{3}\pi r^2 d$.]



Solution: Since the volume depends on two variables, we must eliminate one using similar triangles. As shown, we can relate r and d by $\frac{r}{d} = \frac{2}{3}$, or $r = \frac{2}{3}d$. Using this in V, we get

$$V(d) = \frac{1}{3}\pi \left(\frac{2}{3}d\right)^2 d = \frac{4}{27}\pi d^3.$$

From this we can find $\frac{dV}{dt}$:

$$\frac{dV}{dt} = \frac{dV}{dd} \cdot \frac{dd}{dt} = \frac{4}{9}\pi d^2 \frac{dd}{dt}.$$

We're given that $\frac{dV}{dt} = -1.5 \text{ in}^3/\text{min}$, and that $d = 2$. Thus, plugging in gives

$$-1.5 = \frac{4}{9}\pi (2)^2 \frac{dd}{dt} = \frac{16\pi}{9} \cdot \frac{dd}{dt},$$

which allows us to solve for $\frac{dd}{dt}$ to get

$$\frac{dd}{dt} = -\frac{13.5}{16\pi} \approx -0.26857 \text{ in/min}.$$