8. [12 points] At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing “Hail to the Victors” has a direct impact on the number of points our team scores. If the band plays for \( x \) minutes, then the Wolverines will score

\[
W(x) = -0.48x^2 + 7.2x + 63
\]

points in the game. Assume that the band can play for a maximum of 10 minutes.

a. [5 points] How long should the band play to maximize the number of points Michigan scores? Show your work and explain.

**Solution:** This is a global maximum problem, and we are asked to find the global max of \( W(x) \) over the interval \([0, 10]\). Let’s begin by finding all the critical points. \( W'(x) = -0.96x^2 + 7.2 \), and setting this equal to zero yields \( x = 7.5 \). Now, since \( W(x) \) is an inverted parabola (the coefficient of \( x^2 \) is negative), then the critical point is a local and global maximum. Thus, simply by knowing that we’re dealing with an inverted parabola we are now assured that \( x = 7.5 \) is in fact the location of the global maximum of \( W(x) \) on the interval \([0, 10]\), telling us that the band should play for 7.5 minutes in order for the Wolverines to score the maximum number of points.

b. [5 points] The band affects how many points Ohio State scores as well. When the U-M band plays for \( x \) minutes the Buckeyes score

\[
B(x) = -x^2 + 8x + 84
\]

points in the game. Find the number of minutes the band should play to maximize the margin of victory for Michigan (i.e., the points by which Michigan wins or loses). Again, please show all work.

**Solution:** The margin of victory for Michigan is \( M(x) = W(x) - B(x) = -0.52x^2 - 0.8x - 21 \). The function \( M(x) \) is concave up everywhere (it’s a parabola opening upward), so even if it has a critical point on \([0,10]\), that critical point must be a local minimum. Thus, the global maximum of \( M(x) \) on \([0,10]\) can occur only at one of the endpoints. Checking both endpoints, we see that \( M(10) \) is greater than \( M(0) \), so the global maximum occurs at \( x = 10 \). This tells us that the band should play for 10 minutes to maximize the Wolverine’s margin of victory.

c. [2 points] What will be the score of the game for the case you found in part (b)?

**Solution:**

| Michigan: | 87 points |
| Ohio State: | 64 points |

9. [5 points] —based on your score for the Calculus Concept Inventory.