

8. [12 points] At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing “Hail to the Victors” has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points in the game. Assume that the band can play for a maximum of 10 minutes.

- a. [5 points] How long should the band play to maximize the number of points Michigan scores? Show your work and explain.

Solution: This is a global maximum problem, and we are asked to find the global max of $W(x)$ over the interval $[0, 10]$. Let's begin by finding all the critical points. $W'(x) = -0.96x^2 + 7.2$, and setting this equal to zero yields $x = 7.5$. Now, since $W(x)$ is an inverted parabola (the coefficient of x^2 is negative), then the critical point is a local and global maximum. Thus, simply by knowing that we're dealing with an inverted parabola we are now assured that $x = 7.5$ is in fact the location of the global maximum of $W(x)$ on the interval $[0, 10]$, telling us that the band should play for 7.5 minutes in order for the Wolverines to score the maximum number of points.

- b. [5 points] The band affects how many points Ohio State scores as well. When the U-M band plays for x minutes the Buckeyes score

$$B(x) = -x^2 + 8x + 84$$

points in the game. Find the number of minutes the band should play to maximize the margin of victory for Michigan (*i.e.*, the points by which Michigan wins or loses). Again, please show all work.

Solution: The margin of victory for Michigan is $M(x) = W(x) - B(x) = .52x^2 - 0.8x - 21$. The function $M(x)$ is concave up everywhere (it's a parabola opening upward), so even if it has a critical point on $[0, 10]$, that critical point must be a local minimum. Thus, the global maximum of $M(x)$ on $[0, 10]$ can occur only at one of the endpoints. Checking both endpoints, we see that $M(10)$ is greater than $M(0)$, so the global maximum occurs at $x = 10$. This tells us that the band should play for 10 minutes to maximize the Wolverine's margin of victory.

- c. [2 points] What will be the score of the game for the case you found in part (b)?

Solution:

Michigan: 87 points

Ohio State: 64 points

9. [5 points] —based on your score for the Calculus Concept Inventory.